# Modelling of an explosion impact on a deformable medium with rigid inclusions with quasiconformal mappings methods

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*Abstract*—**The explosive process influence on the environment with the existing impenetrable fixed inclusion is investigated by quasiconformal mappings numerical methods and step-by-step parameterization of the environment and the process characteristics numerical methods. The boundaries of the crater formed by the explosion, pressed and undisturbed areas of soil are determined. Numerical experiments were performed on the basis of the developed algorithm**

**Keywords—explosion process, quasiconformal mappings, numerical methods, mathematical madelling, fixed inclusion, impenetrable inclusion**

## I. Introduction

At the present stage of its technological development, the national economy of Ukraine and many other states is increasingly introducing the use of explosive energy. Explosive processes are used for crushing rocks, in mining, in construction, in particular, underground and semi-underground structures, to provide materials (eg soil) with the necessary engineering properties, to form large cavities. There are several mathematical models of the explosive process, each of which is used to solve a certain class of problems [1 - 5].

A liquid model is widely used to study explosive processes in the soil [3, 4]. Within the framework of this model, a number of works have been written, which determine the boundaries of the crater, pressed and undisturbed soil zones caused by the explosion of one [8, 9] or two [10, 11] charges in isotropic and anisotropic [8] media, set the required explosive force to form the maximum size of the crater, provided that the integrity of these surrounding objects is preserved [11]. The liquid model of the explosion process is also generalized to the spatial case [12]. The above works consider cases when there are no inviolable obstacles in the path of the blast wave, and the environment in which the explosion occurred is close to homogeneous. However, there are a number of cases where there is an immovable object in the path of the blast wave. The purpose of this work is to determine the position of the boundaries of the crater, pressed and undisturbed areas of soil provided the presence in the affected area of a stationary object of known shape and size, and compare the resulting distribution of formed areas with the distribution formed in the soil by a similar explosion inclusions.

## II. Problem statement

In the environment where the explosion is going to take place, we distinguish for consideration some two-connected region $G_z$ $(z = x + iy)$ (Figure 1) bounded by the charge circuit $L_* = \{z : f_*(x, y) = 0\} = \{x + iy : x = x_*(t)$ , $y = y_*(t)$ , $\alpha_* < t < \beta_*\}$ and some external circuit $L^* = \{z : f^*(x, y, \eta) = 0\} = \{x + iy : x = x^*(t,\eta)$ , $y = y^*(t,\eta)$ , $\alpha^* < t < \beta^*\}$. Also the area occupied by inclusion is specified: $G_z^* \subset G_z$. It's bounded by the circuit $L_0 = \{z : f_0(x, y) = 0\}$ $= \{x + iy : x = x_0(t)$ , $y = y_0(t)$ , $\alpha_0 < t < \beta_0\}$. Here $x_*(t)$, $y_*(t)$, $x_0(t)$, $y_0(t)$, $x^*(t,\eta)$ and $y^*(t,\eta)$ are predefined continuous-differential functions; $\eta$ is the parameter characterizing the position of the outer circuit of the considered area, which will be specified in the process of solving the problem under the condition of stabilization, as described in [9]. We assume that the barrier is absolutely rigid and impermeable to soil particles. So we obtain some three-connected domain $G_z^0 = G_z / G_z^*$.

The process of motion of particles of the medium caused by the explosion action is modeled using the law of motion $\vec{\upsilon} = k\ grad\,\varphi$ and the continuity equation $div\,\vec{\upsilon} = 0$ (similarly to [8, 9]). Here $\vec{\upsilon} = (\upsilon_x(x, y), \upsilon_y(x, y))$ is the particle velocity at a point $(x, y)$, $\varphi = \varphi(x, y) = -P / \rho$ is the potential of the field formed by the explosion, where $\rho$ is the density of the medium, $P$ is the pressure pulse that exerts a charge on the particles of the medium during the explosion, $k = k(|grad\,\varphi|)$ is the socalled fictitious permeability coefficient, which characterizes the ability of particles to break away [8].

The inverse effect of the explosion process characteristics (quasipotential gradient) on the environment characteristics (as, for example, in [8]) is characterized by the formula

$$k \;=k_0+\frac{1}{2}\beta\;\left(I-I^*\right)\left(\left(I-I^0\right)+\left|I-I^0\right|\right)\;, \tag{1}$$

where $k_0=k^0$ the initial distribution of the fictitious permeability coefficient of the medium, $I=\sqrt{\varphi_x^2+\varphi_y^2}=\left|grad\;\varphi\right|$ is the magnitude of the quasi-potential gradient, $I^0$ and $I^*$ are the critical values used to establish the boundaries of the formed sections of the crater, pressed and undisturbed soil zones, the parameter $\beta$ depends on soil type and is determined experimentally.

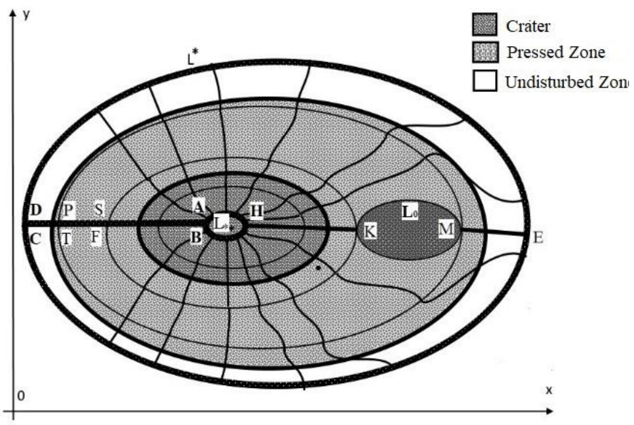


Figure 1. Schematic representation of the investigated area

By introducing a function $\psi=\psi\left(x,y\right)$ that is complex conjugate to $\varphi=\varphi(x,y)$ and forming a conditional section $\Gamma$ of a region $G_z^0$ passing through a certain fixed point $A\in L_*$ along the corresponding desired flow line, we arrive at the problem of quasiconformal mapping $\omega=\omega\left(z\right)=\varphi\left(x,y\right)+\mathrm{i}\,\psi\left(x,y\right)$ of the investigated physical domain $\tilde{G}_z=G_z^0\setminus\Gamma$ to its corresponding quasicomplex potential domain $G_\omega=\left\{\omega=\varphi+i\psi:\,\varphi_*<\varphi<\varphi^*\;0<\psi<Q\right\}$ $\setminus\left\{\omega:\psi=\psi^*,\underline{\varphi}<\varphi<\overline{\varphi}\right\}$ [6, 7]. Since the domain $G_z^*$ is impermeable, one of the flow lines (denote it $\psi(x,y)=\psi^*$) will "circumflex" it, bifurcating at some critical point $K$ ($K\in L_0$) (denoting the potential value in it $\underline{\varphi}=\varphi(K)$), and reconnecting at another critical point $M$ ($M\in L_0$). The value of the potential at this point is similarly denoted by $\overline{\varphi}=\varphi(M)$ $(\varphi_*<\underline{\varphi}<\overline{\varphi}<\varphi^*)$ (Figure 1).

The problem is to solve a system of equations of the Cauchy-Riemann type:

$$\left.\begin{aligned}\kappa\left(|\,grad\varphi)\,|\right)\frac{\partial\varphi}{\partial x}&=\frac{\partial\psi}{\partial y},\\ \kappa\left(|\,grad\varphi)\,|\right)\frac{\partial\varphi}{\partial y}&=-\frac{\partial\psi}{\partial x},\end{aligned}\right.\quad \left(x,y\right)\in\tilde{G}_z\;, \tag{2}$$

under boundary conditions $\varphi\big|_{L_*}=\varphi_*$, $\varphi\big|_{L^*}=\varphi^*$, $\psi\big|_{AD}=0$, $\psi\big|_{BC}=Q=\oint\limits_{L_*}-\upsilon_y dx+\upsilon_x dy$, $\psi\big|_{\underline{HKME}}=\psi\big|_{\overline{HKME}}=\psi_*$. The values $\underline{\varphi}$, $\overline{\varphi}$, $\psi^*$, $Q$ and $\eta$ will be identified iteratively in the process of problem solving.

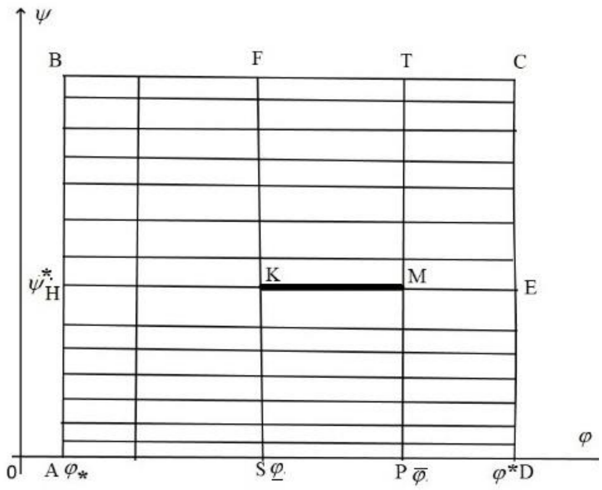


Figure 2. Schematic representation of the corresponding quasicomplex potential domain

Given the geometric complexity of the physical domain $\tilde{G}_z$, we turn to the solution of the inverse problem on the quasiconformal mapping of the domain of quasicomplex potential $G_\omega$ to the domain $\tilde{G}_z$ (as described, for example, in [11]), which is reduced to solving a system of equations:

$$\frac{\partial}{\partial\varphi}\left(\frac{1}{\kappa\left(\dfrac{1}{J}\sqrt{(\dfrac{\partial y}{\partial\psi})^2+(\dfrac{\partial x}{\partial\varphi})^2}\right)}\frac{\partial x}{\partial\varphi}\right)+$$

$$+\frac{\partial}{\partial\psi}\left(\kappa\left(\frac{1}{J}\sqrt{(\frac{\partial y}{\partial\psi})^2+(\frac{\partial x}{\partial\varphi})^2}\right)\frac{\partial x}{\partial\psi}\right)=0.$$

$$\frac{\partial}{\partial\varphi}\left(\frac{1}{\kappa\left(\frac{1}{J}\sqrt{(\frac{\partial y}{\partial\psi})^2+(\frac{\partial x}{\partial\varphi})^2}\right)}\frac{\partial y}{\partial\varphi}\right)+$$

$$+\frac{\partial}{\partial\psi}\left(\kappa\left(\frac{1}{J}\sqrt{(\frac{\partial y}{\partial\psi})^2+(\frac{\partial x}{\partial\varphi})^2}\right)\frac{\partial y}{\partial\psi}\right)=0,$$

$$J=\frac{\partial x}{\partial\varphi}\frac{\partial y}{\partial\psi}-\frac{\partial x}{\partial\psi}\frac{\partial y}{\partial\varphi},\ (\varphi,\psi)\in G_\omega \quad (3)$$

under the boundary conditions

$$\begin{cases} x_*{}'_t(t)\dfrac{\partial y(\varphi_*,\psi)}{\partial\varphi}+y_*{}'_t(t)\dfrac{\partial x(\varphi_*,\psi)}{\partial\varphi}=0, \\ \alpha_*<t<\beta_*, \\ x^*{}'_t(t)\dfrac{\partial y(\varphi^*,\psi)}{\partial\varphi}+y^*{}'_t(t)\dfrac{\partial x(\varphi^*,\psi)}{\partial\varphi}=0, \\ \alpha^*<t<\beta^*,. \\ x_0{}'(t)\dfrac{\partial y(\varphi,\psi^*+0)}{\partial\psi}+y_0{}'(t)\dfrac{\partial x(\varphi,\psi^*+0)}{\partial\psi}=0, \\ \alpha_0<t<\beta_0, \\ x_0{}'(t)\dfrac{\partial y(\varphi,\psi^*-0)}{\partial\psi}+y_0{}'(t)\dfrac{\partial x(\varphi,\psi^*-0)}{\partial\psi}=0, \\ \alpha_0<t<\beta_0, \end{cases} \quad (4)$$

$$\begin{aligned} &x(\varphi_*,\psi)=x_*(t_*(\psi)),\ \ y(\varphi_*,\psi)=y_*(t_*(\psi)),\\ &x(\varphi^*,\psi)=x^*(t^*(\psi)),\ \ y(\varphi^*,\psi)=y^*(t^*(\psi)), \end{aligned} \quad (5)$$

$$0\le\psi\le Q,$$

$$x(\varphi,\psi_*)=x_0(t_0(\varphi)),\ \ y(\varphi,\psi_*)=y_0(t_0(\varphi)),\ \underline{\varphi}\le\varphi\le\overline{\varphi}. \quad (6)$$

$$\begin{aligned} x(\varphi,0)=x(\varphi,Q),\\ y(\varphi,0)=y(\varphi,Q), \end{aligned}\quad \varphi_*\le\varphi\le\varphi^*. \quad (7)$$

Here $t=t_*(\psi)$, $t=t_*(\psi)$ and $t=t_0(\varphi)$ are the functions that determine the dependence of the parameters $t$ of the boundary setting on the values of the functions of potential and flow on the curves $L_*$, $L^*$ and $L_0$, accordingly.

## III. DIFFERENCE ANALOGUE AND ALGORITHM OF NUMERICAL SOLVING OF THE PROBLEM

The rectangular quasicomplex potential domain $G_\omega$ is divided into 6 rectangular subdomains: $G_\omega^1=\{(\varphi,\psi):\varphi_*\le\varphi\le\underline{\varphi},\ 0\le\psi\le\psi^*\}$, $G_\omega^2=\{(\varphi,\psi):\varphi_*\le\varphi\le\underline{\varphi},\ \psi^*\le\psi\le Q\}$, $G_\omega^3=\{(\varphi,\psi):\underline{\varphi}\le\varphi\le\overline{\varphi},\ 0\le\psi\le\psi^*\}$, $G_\omega^4=\{(\varphi,\psi):\underline{\varphi}\le\varphi\le\overline{\varphi},\ \psi^*\le\psi\le Q\}$, $G_\omega^5=\{(\varphi,\psi):\overline{\varphi}\le\varphi\le\varphi^*,\ 0\le\psi\le\psi^*\}$ and $G_\omega^6=\{(\varphi,\psi):\overline{\varphi}\le\varphi\le\varphi^*,\ \psi^*\le\psi\le Q\}$ ( $\partial G_\omega^1=AHKF$ , $\partial G_\omega^2=HBSK$ , $\partial G_\omega^3=FKMT$ , $\partial G_\omega^4=KSPM$ , $\partial G_\omega^5=TMED$ and $\partial G_\omega^6=MPCE$ ). Then we give a uniform partition $n_1\times m_1$, $n_1\times m_2$, $n_2\times m_1$, $n_2\times m_2$, $n_3\times m_1$, $n_3\times m_2$ of subdomains $G_\omega^1$, $G_\omega^2$, $G_\omega^3$, $G_\omega^4$, $G_\omega^5$ and $G_\omega^6$, respectively. In this case, the domain $G_\omega=\{(\varphi,\psi):\varphi_*\le\varphi\le\varphi^*,\ 0\le\psi\le Q\}$ ( $\partial G_\omega=ABCD$ ) will be divided into $n\times m$ nodes ( $n=n_1+n_2+n_3$, $m=m_1+m_2$ ), which are defined as follows:

$$\varphi_i=\begin{cases}\varphi_*+i\Delta\varphi_1,\ \Delta\varphi_1=\dfrac{\underline{\varphi}-\varphi_*}{n_1},\ i=\overline{1,n_1},\\ \underline{\varphi}+(i-n_1)\Delta\varphi_2,\ \Delta\varphi_2=\dfrac{\overline{\varphi}-\underline{\varphi}}{n_2},\ i=\overline{n_1+1,n_2},\\ \overline{\varphi}+(i-n_1-n_2)\Delta\varphi_3,\ \Delta\varphi_3=\dfrac{\varphi^*-\overline{\varphi}}{n_3},\\ i=\overline{n_1+n_2+1,n_3}\end{cases} \quad (8)$$

$$\psi_j=\begin{cases} j\Delta\psi_1,\ \Delta\psi_1=\dfrac{\psi^*}{m_1},\ j=\overline{1,m_1},\\ (j-m_1)\Delta\psi_2,\ \Delta\psi_2=\dfrac{Q-\psi^*}{m_2},\ j=\overline{m_1+1,m_2},\end{cases} \quad (9)$$

The steps of the grid domains $\Delta\varphi_1$, $\Delta\varphi_2$, $\Delta\varphi_3$, $\Delta\psi_1$, $\Delta\psi_2$ and the number of partition nodes $n_1$, $n_2$, $n_3$, $m_1$, $m_2$ will be specified iteratively in the process of solving the problem together with the total flow $Q$ and critical values $\underline{\varphi}$, $\overline{\varphi}$, $\psi^*$ as follows: $Q=m_1\Delta\psi_1+m_2\Delta\psi_2$ , $\underline{\varphi}=\dfrac{\varphi_*+\underline{a}\varphi^*}{1+\underline{a}}$ , $\overline{\varphi}=\dfrac{\varphi^*+\overline{a}\varphi_*}{1+\overline{a}}$ , where $\underline{a}=\dfrac{\gamma_{11}(n_1+1)}{\gamma_{21}(n_2-1)+\gamma_{31}(n_3+1)}+$ $+\dfrac{\gamma_{12}(n_1+1)}{\gamma_{22}(n_2-1)+\gamma_{32}(n_3+1)}$ , $\overline{a}=\dfrac{\gamma_{31}(n_3+1)}{\gamma_{12}(n_2-1)+\gamma_{11}(n_1+1)}+$ $+\dfrac{\gamma_{32}(n_3+1)}{\gamma_{22}(n_2-1)+\gamma_{21}(n_3+1)}$ (as, for example, in [11]) .

The difference analogue of problem (3) - (7) will be approximated by homogeneous conservative difference schemes:

$$\frac{x_{i+1,j}-x_{i,j}}{k_{i+1/2,j}}-\frac{x_{i,j}-x_{i-1,j}}{k_{i-1/2,j}}+\gamma^2{}_{p,s}(k_{i,j+1/2}(x_{i,j+1}-x_{i,j})+$$
$$+k_{i,j-1/2}(x_{i,j}-x_{i,j-1}))=0. \qquad (10)$$
$$\frac{y_{i+1,j}-y_{i,j}}{k_{i+1/2,j}}-\frac{y_{i,j}-y_{i-1,j}}{k_{i-1/2,j}}+\gamma^2{}_{p,s}(k_{i,j+1/2}(y_{i,j+1}-y_{i,j})+$$
$$+k_{i,j-1/2}(y_{i,j}-y_{i,j-1}))=0.$$

Here $x_{i,j}=x(\varphi_i,\psi_j)$, $y_{i,j}=y(\varphi_i,\psi_j)$, $k_{i+1/2,j}=\frac{k_{i+1,j}+k_{i,j}}{2}$, $k_{i-1/2,j}=\frac{k_{i,j}+k_{i-1,j}}{2}$, $k_{i,j+1/2}=\frac{k_{i,j+1}+k_{i,j}}{2}$, $k_{i,j-1/2}=\frac{k_{i,j}+k_{i,j-1}}{2}$, $\gamma_{p,s}=\frac{\Delta\varphi_p}{\Delta\psi_s}$, $p=\overline{1,3}$, $s=\overline{1,2}$.

$$\begin{cases} x_*{}'_t(t_*(\psi_j))\dfrac{y_{1,j}-y_{0,j}}{\Delta\varphi_1}+ \\ +y_*{}_t(t_*(\psi_j))\dfrac{x_{1,j}-x_{0,j}}{\Delta\varphi_1}=0,\ j=\overline{0,m}, \\ x^*{}'_t(t^*(\psi_j))\dfrac{y_{n,j}-y_{n-1,j}}{\Delta\varphi_3}+ \\ +y_*{}_t(t^*(\psi_j))\dfrac{x_{n,j}-x_{n-1,j}}{\Delta\varphi_3}=0,\ j=\overline{0,m}, \\ x_0{}'_t(t_0(\varphi_i))\dfrac{y_{i,m_1}-y_{i,m_1-1}}{\Delta\psi_1}+ \\ +y_0{}'_t(t_0(\varphi_i))\dfrac{x_{i,m_1}-x_{i,m_1-1}}{\Delta\psi_1}=0, i=\overline{n_1+1,n_1+n2}, \\ x_0{}'_t(t_0(\varphi_i))\dfrac{y_{i,m_1+1}-y_{i,m_1}}{\Delta\psi_2}+ \\ +y_0{}'_t(t_0(\varphi_i))\dfrac{x_{i,m_1+1}-x_{i,m_1}}{\Delta\psi_2}=0, i=\overline{n_1+1,n_1+n2} \end{cases} \qquad (11)$$

$$\begin{matrix} x_{0,j}=x_*\left(t_*\left(\psi_j\right)\right),\ y_{0,j}=y_*\left(t_*\left(\psi_j\right)\right), \\ x_{n,j}=x^*\left(t^*\left(\psi_j\right)\right),\ y_{n,j}=y^*\left(t^*\left(\psi_j\right)\right), \end{matrix}\quad j=\overline{0,m}, \qquad (12)$$

$$x_{i,m_1}=x_0\left(t_0\left(\varphi_i\right)\right),\ y_{i,m_1}=y_0\left(t_0\left(\varphi_i\right)\right),\quad i=\overline{n_1,n_2}. \qquad (13)$$

$$\begin{matrix} x_{i,0}=x_{i,m}, \\ y_{i,0}=y_{i,m}, \end{matrix}\quad i=\overline{0,n}. \qquad (14)$$

We will solve the problem according to the algorithm of alternate parameterization of the characteristics of the environment and process developed by the authors earlier [10].

Note that the described option of transition from a three-connected region to a single-connected one is only one of the possible variants. Its choice is determined by the form and position of inclusion.

## CONCLUSION

As a result of solving the problem, the boundaries of the section of the crater, pressed and undisturbed areas of soil, as well as the characteristic flow line were obtained.

Based on the developed algorithm, a number of numerical experiments were performed, which show that the presence of fixed impermeable inclusions in the soil affects the distribution of the explosion zones formed by the explosion: crater pressed and undisturbed areas of the soil. The form of inclusion also matters.

In the future - the study of the impact of the explosion on the environment in the presence of two or more inclusions, the stability of inclusions under the action of the explosion, the impact on the distribution of the formed zones of anisotropy of the environment, the corresponding spatial problems.

## REFERENCES

[1] V. Kravets, V. Korobyichuk, and V. V. Boiko, Physical Processes of Applied Geodynamics of an Explosion. Zhytomyr : ZSTU, 2015.

[2] V. Korobijchuk, R. Sobolevs'kyj, and O. Zubchenko, “Investigation of ways to minimize the cost of drilling and blasting of blocks of decorative stone”, Visnyk Zhytomyrs'kogo Derzhavnogo Tehnologichnogo Universytetu, serie “Tehnichal sciences”, no. 4 (39), pp. 301–308, 2006.

[3] A. Bomba, V. Bulavatskii, V. Skopetskii, Nonlinear mathematical models of geohidrodynamic processes, Kyiv: Naukova Dumka, 2007.

[4] V. Bulavatskii, and I. Luchko, “Some inverse problems of the pulsed-hydrodynamic theory of explosion on the discharge”, In Investigations on Boundary Value Problems of Hydrodynamics and Thermophysics, Kiev, 1979, pp. 53–64. V. M. Bulavatskii, Yu. G. Kryvonos, and V. V. Skopetskii, Nonclassic mathematical models of heat- and mass transfer processes, Kyiv: Naukova Dumka, 2005.

[5] O. E. Vlasov, S. A. Smyrnov, “About explosion modelling”, Explosion business, Moskva: Nedra, no. 59, pp. 109 – 117, 1966.

[6] David E. Blair, Inversion theory and conformal mapping, American Mathematical Society, 2000.

[7] James Nearling, Mathematical tools for physics, Miami, 2008.

[8] A. Bomba, K. Malash, “ Modeling of the explosion process in an anisotropic medium with quasiconformal mapping methods”, Transactions of Kremenchuk Mykhailo Ostrohradskyi National University, no. 4 (105), pp. 28–33, 2017.

[9] A. Bomba, and K. Malash. “Modeling of explosive processes in anisotropic media where boundary of the influence region is identified”, Mathematical and Computer Modelling, serie “Technical sciences”, no. 18, pp. 3–16, 2018.

[10] A. Bomba, and K. Malash, “Modeling the formation of craters caused by the two charges explosion using quasiconformal mappings numerical methods” in Advanced Computer Information Technologies: 9th International Conference Proceedings, Ceske Budejovice, Czech Republic, 5-7 June, 2019. pp.: 113 – 116.

[11] A. Bomba, A. Safonyk, and K. Malash, “Optimization of controlled explosion processes parameters using complex analysis methods”, Automatyka Pomiary w Gospodarce i Ochronie Środowiska, no. 9(1), pp. 29–32, 2019.

[12] K. Malash, and A. Bomba, “Spatial generalization the explosion process mathematical model using quasiconformal mappings methods” Tranzactions of Kharkiv Politechnical National University. Serie: Mathematical Modelling in the Science and Technilogies, no. 8 (1333), pp. 138–143, 2019.