Model of redistribution of political parties supporters

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Abstract – **The goal of this research is to create a mathematical model of political parties supporters redistribution. In this work it is done via creating a system of differential equation for the case when society has two political parties and non-partisan citizens. In the second part, the model is generalized for the case of n political parties or groups.**

Keywords – mathematical model; society; party; party supporters; differential equations;

I. Introduction

The problem of prognostication of political parties development becomes more and more actual in modern society. Success of a political party highly depends on how much the party is supported by society members. However, this problem doesn't have much of mathematical development. In this work an attempt to describe redistribution dynamic of political parties and groups supporters was made.

Relatively close to this problem are classic models of different biological species population dynamics: predator-prey model (also known as Lotka-Volterra model) and model of two biological species competition [1-6]. Predator-prey model describes how changes the population of prey, which have unlimited resources but are consumed by predators, and predators, which have a limited resources of their prey. Closer to the problem of political parties supporters is the model of two biological species competition, where they do not hunt each other directly, but have a competition for a common limited resource. Also logistic growth curve [7] might be helpful in order to apply a limit to a population growth.

II. Mathematical model

Let's consider a society which has only two political parties A and B. Non-partisan people is denoted as C. Let's set, that the society has a problem and a suggestion on how to solve it. Each political party expresses own opinion regarding the proposed solution (furthermore just opinion). Let's set, that an opinion of ultimate support of the solution equals 1 and an opinion of very strong objection equals -1, where 0 is a neutral attitude to the solution. Also each political party agitates for own opinion. Because of such agitation some members of the society might change their opinions and also the party which they support. Let's put that if a citizen gets agitated by another person, he or she changes own opinion toward the agitator opinion. However, the value of the influence on the person's opinion is not a constant and depends on the opinions of both sides in the conversation. Obviously, that a person who has a strong opinion (absolute value of the opinion is close to 1) does quite a big influence on his interlocutor, whereas a person with weak opinion (opinion absolute value is close to zero) does very small influence. Moreover, it is quite hard to change political position of a person which has a strong opinion, however it is easy to agitate and make a person with weak political position change their mind. To represent such relationship, the following influence formula is proposed:

$$f(x,y) = \frac{|x-y|}{2} \cdot \frac{cos(\pi y)+1}{2} \cdot \left(1 - \frac{cos(\pi x)+1}{2}\right) \qquad (1)$$

where:

f – is the influence value,
x – is the opinion value of the agitator,
y – is the opinion value of the side which changes its opinion.

Further, influence value of x to y we will denote as x_y.

Let's denote number of supporters of parties A and B as a and b correspondingly and c as the number of non-partisan society members. Also let's say, that α is the opinion of party A, and β is the opinion of party B. As non-partisan people are neutral, let's set their opinion γ to 0.

To describe how changes the number of each party supporters in time, let's create a system of differential equations. First, let's take a closer look at how changes the number of supporters of a single party, say, party A. Because of party A agitation it gets new supporters. There are two sources of newcomers for party A: ex-members of party B and non-partisan people C. This could be formally written as:

$$\frac{da}{dt} = \left(\alpha_\beta b + \alpha_\gamma c\right) a \qquad (2)$$

From the other side, members of party B don't stay silent, but in their own turn agitate members of party A. As the result of that, party A loses some of its supporters:

$$\frac{da}{dt} = -\beta_\alpha ba \qquad (3)$$

If put together (2) and (3), we'll get the equation which describes number of supporters of party A in time:

$$\frac{da}{dt} = \left(\alpha_\beta b + \alpha_\gamma c - \beta_\alpha b\right)a \qquad (4)$$

The equation for supporters of party B will be symmetric and could be obtained using the same idea:

$$\frac{db}{dt} = \left(\beta_\alpha a + \beta_\gamma c - \alpha_\beta a\right)b \qquad (5)$$

According to the conditions of the current problem, non-partisan people are neutral, so their quantity only decreases in time:

$$\frac{dc}{dt} = -\left(\alpha_\gamma a + \beta_\gamma b\right)c \qquad (6)$$

For simplicity, let's set that the number of people in the society doesn't change, which means:

$$a + b + c = const \qquad (7)$$

Or formula (7) could be rewritten in differential form:

$$\frac{da}{dt} + \frac{db}{dt} + \frac{dc}{dt} = 0 \qquad (8)$$

Finally, let's put everything (4), (5), (6) and (8) together:

$$\begin{cases} \frac{da}{dt} = \left(\alpha_\beta b + \alpha_\gamma c - \beta_\alpha b\right)a \\ \frac{db}{dt} = \left(\beta_\alpha a + \beta_\gamma c - \alpha_\beta a\right)b \\ \frac{dc}{dt} = -\left(\alpha_\gamma a + \beta_\gamma b\right)c \\ \frac{da}{dt} + \frac{db}{dt} + \frac{dc}{dt} = 0 \end{cases} \qquad (9)$$

where:

a – number of supporters of party A,
b – number of supporters of party B,
c – number of non-partisan members of the society,
α – opinion of party A,
β – opinion of party B,
γ – opinion of non-partisan, is equal to 0,
$\alpha_\beta, \alpha_\gamma, \beta_\alpha, \beta_\gamma$ – constants which describe influences between society groups. Calculated from values of α, β and γ using formula (1).

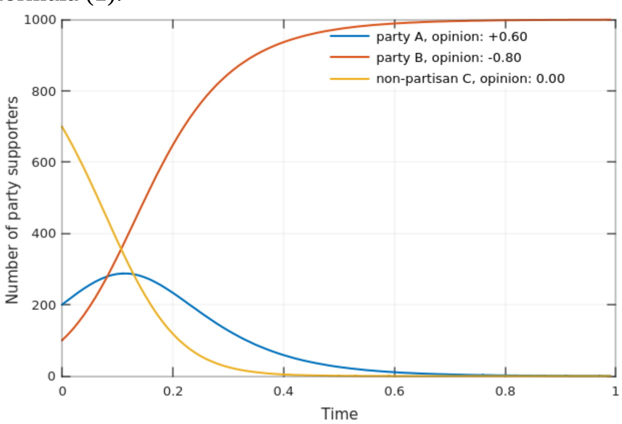

Figure 1. Solution of the system (9) when $a_0 = 200, b_0 = 100, c_0 = 700$ and $\alpha = 0.6, \beta = -0.8$

To solve the obtained system of differential equations (9) we need to set initial conditions first. Let a_0 and b_0 are numbers of supporters of parties A and B correspondingly on the first step, c_0 is the number of non-partisans. Parties opinions α and β are parameters. According to the conditions, non-partisan people are neutral, so $\gamma = 0$.

Let's set $a_0 = 200$, $b_0 = 100$, $c_0 = 700$. On Fig. 1 is shown solution for $\alpha = 0.6, \beta = -0.8$. Fig. 2 shows solution for $\alpha = 0.6, \beta = -0.6$

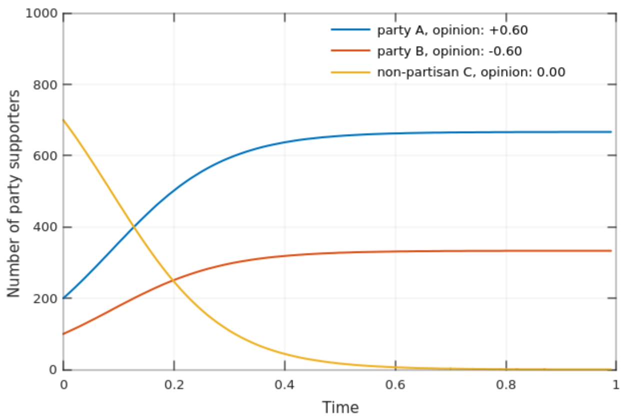

Figure 2. Solution of the system (9) when $a_0 = 200, b_0 = 100, c_0 = 700$ and $\alpha = 0.6, \beta = -0.6$

III. Model generalization

However, non-partisan members of the society might have opinion different from neutral ($\gamma \neq 0$). In other words, society might already have an opinion regarding the discussed problem. In such case non-partisan people will give some influence on political parties too. Moreover, they support each other regarding their opinion, which is very similar to what a political party does. As the result of that, system of differential equations (9) becomes:

$$\begin{cases} \frac{da}{dt} = \left(\alpha_\beta b + \alpha_\gamma c - \beta_\alpha b - \gamma_\alpha c\right)a \\ \frac{db}{dt} = \left(\beta_\alpha a + \beta_\gamma c - \alpha_\beta a - \gamma_\beta c\right)b \\ \frac{dc}{dt} = \left(\gamma_\alpha a + \gamma_\beta b - \alpha_\gamma a - \beta_\gamma b\right)c \\ \frac{da}{dt} + \frac{db}{dt} + \frac{dc}{dt} = 0 \end{cases} \qquad (10)$$

Obtained system of differential equations (10) can also be generalized for the case of $n (n \geq 2)$ parties (political groups):

$$\begin{cases} \frac{dx_i}{dt} = \left(\sum_j^n f(p_i, p_j)x_j - \sum_j^n f(p_j, p_i)x_j\right)x_i, j \neq i; i, j = \overline{1, n} \\ \sum_i^n \frac{dx_i}{dt} = 0 \end{cases}$$

(11)

where:

n – number of political parties (groups),
X – vector of political parties (groups),
p – vector of opinions of each political party (group),
x – vector of supporters number of each political party (group),

$f(p_i, p_j)$ – influence of party (group) X_i on opinion of party (group) X_j supporters. Calculated by (1),

$i, j = \overline{1, n}$.

System of differential equations (11) also might be used for modeling of the society in which non-partisan people are divided by opinions into several groups. In case of presence of political parties it is just required to mark k elements of X as non-partisan. Obviously, absolute value of opinion for non-partisan groups is usually lower than opinion of a political party.

IV. Conclusions

Model of redistribution of political parties (groups) supporters is proposed. It takes into account the presence of non-partisan people in the society, who gets agitated by political parties and groups as well as the situation when people change their mind and become supporters of another party. The model was generalized for case of n political parties and also could be used for modeling of the situation when beside political parties some non-patisan groups with own opinion already present in the simulated society.

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