Mathematical modeling of water purification in a bioplato filter

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***Abstract* — A mathematical model of filtration taking into account clogging and suffusion in the bioplato filter system in the two-dimensional case was built. The constructed mathematical model takes into account the physical effects of the dynamic change of porosity and the dependence of the filtration coefficient on the concentration of contaminants, which is not in the known analogues**

***Keywords* — bioplato filter; filtration problem; clogging; suffusion; finite element method.**

I. Introduction

In recent years, bioplato have become widespread and are used for treatment and re-treatment of domestic, industrial wastewater, surface runoff in different countries [2, 3]. This is due to the fact that they require virtually no consumption of electricity and chemical reagents, significant maintenance and provide the necessary quality of water treatment from a wide range of pollutants of organic and mineral nature [1].

However, in the known structures of the bioplato there is a gradual clogging of the pore space of the filter backfill and bottom drainage with biofilm and mineralized sludge, accumulation of sludge in the bottom of structures, reduced oxygen supply to the root system of plants, which can lead to reduced efficiency . Especially negative is the supply of water to such bioplato with a high content of suspended particles, which significantly enhances the negative processes that take place in the thickness of the filter backfill.

In essence, it is necessary to study the process of migration of undissolved particles in porous media. The importance of the process of migration of solid particles in porous media is emphasized in [5], because along with them can migrate and viruses associated with these particles. A mathematical model of suspension transfer in porous media was constructed and applied to suffusion processes in [6]. In [6] the transfer of iron nanoparticles in porous media and their use for purification of contaminated soil and groundwater was investigated. However, in the process of clogging there is a dynamic change in the porosity of the porous medium, which, in turn, affects the filtration coefficient and, indirectly, the entire filtration process.

II. Mathematical Model of Water Purification in a Bioplato Filter

A bioplato filter with a length of 50 meters and a height of 2 meters with a gravel backfill with a particle size of 20 mm was considered as a model problem. Contaminated water is fed through the upper drainage system from perforated pipes and is removed in the lower part of the backfill by means of perforated drainage located at the bottom of the bioplato.

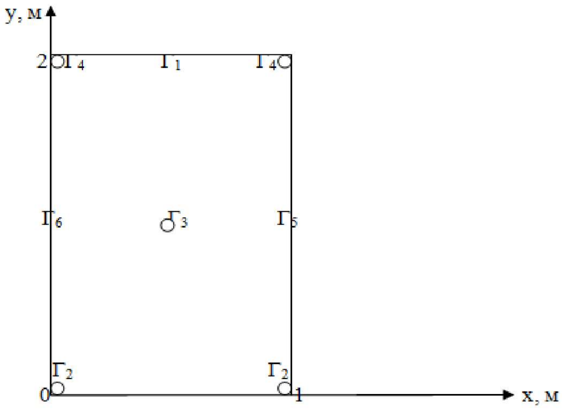

Figure. 1. Cross section of the solution area of the problem

From [4] the mathematical model of filtration taking into account clogging contains the following equations

$$\nabla\cdot(\rho_p(c)k_h(c,\sigma)\nabla h)=\sigma\frac{\partial\rho_p}{\partial c}\cdot\frac{\partial c}{\partial t}-\frac{\rho_p}{\rho_s}\frac{\partial s}{\partial t},X\in\Omega, \quad (1)$$

$$\sigma\left(1-\frac{c}{\rho_p}\frac{\partial\rho_p}{\partial c}\right)\frac{\partial c}{\partial t}= \nabla\cdot(D_c\nabla c)-u\left(1-\frac{c}{\rho_p}\frac{\partial\rho_p}{\partial c}\right)\nabla c-\frac{\partial s}{\partial t},\ X\in\Omega, \quad (2)$$

$$\frac{\partial s}{\partial t}=\alpha\cdot c-\beta\cdot s,\ X\in\Omega, \quad (3)$$

$$u=-k_h(c,s,\sigma)\nabla h, X\in\Omega. \quad (4)$$

Mathematical model (1) – (4) allowed to increase the adequacy of the results for the studied physical processes, but does not reduce the negative impact of clogging and showed the need to develop engineering solutions to reduce the impact of clogging-suffusion processes on filtration processes. This will require, in turn, the consideration of these engineering solutions in the mathematical model. That is why in this paper the authors propose to supplement the bioplato filter system with an additional system of perforated pipes, which is located in the thickness of the filter backfill. The additional system of perforated pipes sets the conditions for pumping water, but with less intensity. This water is re-fed to the upper system of perforated pipes. Thus, the filter water is purified in two stages.

Initial conditions:

$$c(x,y,0) = C_0, \ (X) \in \overline{\Omega},$$

$$s(x,y,0) = s_0, \ (X) \in \overline{\Omega}.$$

Boundary conditions

$$h(\mathbf{X},t)\big|_{\mathbf{X}\in\Gamma_1,\Gamma_4} = y.$$

At the boundaries and the flow depends on the distance to the right edge:

$$q\big|_{X\in\Gamma_2} = \frac{q_{min}(x - x_2)}{x_1 - x_2} + \frac{q_{max}(x - x_1)}{x_2 - x_1},$$

$$q\big|_{X\in\Gamma_3} = w * q\big|_{X\in\Gamma_2},$$

where

w – coefficient,

q – fluid flow per unit length,

$x_1 = 0$,

$x_2 = 1$,

$c(X,t)\big|_{X\in\Gamma_1,\Gamma_4} = C_{max}$,

Γ_5, Γ_6 – limits of impermeability.

Here $\sigma(X,t)$ – the porosity of the soil, which is variable over time due to changes in the concentration of clogging particles; $s(X,t)$ – mass concentration of clogging particles (mass of particles that are associated with the soil skeleton and are classified per unit volume); c – concentration of the suspension to be filtered (mass of suspended particles per unit volume of pore liquid); ρ_s – the density of the material of clogging particles; $\rho_p = \rho_p(c)$ – the density of the pore liquid (suspension), which depends on the concentration of suspended particles; $k_h = k_h(c,\sigma)$ – filtration coefficient, which depends on the concentration of the suspension and porosity; h – pressure in the pore fluid; D_c – particle dispersion coefficient in the pore suspension; $u = -k_h(c,s,\sigma)\nabla h$ – filtration rate of the pore suspension; α – particle adhesion rate coefficient; β – particle separation rate coefficient.

The finite element method was used to find an approximate solution of the boundary value problem.

The weak formulation of the boundary value problem is as follows. Multiply equation (1) by the test function

$$v1(X) \in H_0 = \{v1(X) : v1(X) \in W_2^1(\Omega)\}, v1(X)\big|_{\Gamma_1 \cup \Gamma_2} = 0,$$

integrate over the domain, apply the Ostrogradsky-Gauss formula and obtain

$$\iint_\Omega (k_h \rho_p \nabla h \nabla v1) d\Omega + \iint_\Omega (\sigma \frac{\partial \rho_p}{\partial c} \cdot \frac{\partial c}{\partial t} \cdot v1) d\Omega - \iint_\Omega (\frac{\rho_p}{\rho_s} \cdot \frac{\partial s}{\partial t} \cdot v1) d\Omega = 0,$$

Multiply equation (2) and integrate the initial condition for the concentration of the suspension on the test function by region

$$v2(X) \in H_0 = \{v2(X) : v2(X) \in W_2^1(\Omega)\}, v2(X)\big|_{\Gamma_1} = 0,$$

apply the Ostrogradsky-Gauss formula and obtain

$$\iint_\Omega (D_c \nabla c \nabla v2) d\Omega - \iint_\Omega (u(1 - \frac{c}{\rho_p}\frac{\partial \rho_p}{\partial c}) \cdot \nabla c \cdot v2) d\Omega + \iint_\Omega (\frac{\partial s}{\partial t} \cdot v2) d\Omega =$$

$$\iint_\Omega (\sigma(1 - \frac{c}{\rho_p}\frac{\partial \rho_p}{\partial c}) \cdot \frac{c^i - c^{i-1}}{dt} \cdot v2) d\Omega,$$

$$\iint_\Omega c(x,y,0) \cdot v2(X) d\Omega = \iint_\Omega c_0 \cdot v2(X) d\Omega,$$

$$u = -k_h \nabla h, \ s^i = (\alpha \cdot c^{i-1} - \beta \cdot s^{i-1}) \cdot dt + s^{i-1}.$$

In order to find an approximate generalized solution of the obtained problem, time sampling should be used (for more detailed information, see [4]).

III. Conclusions and Directions of Further Research

An improved mathematical model of the suspension filtration process in a bioplato filter is constructed, which, unlike known analogues, takes into account the nonlinear dependence of the filtration coefficient on the concentration of clogging particles and the dynamic change of porosity in the filtration process. The presented mathematical model also takes into account the presence of an additional system of perforated pipes, which is located in the thickness of the filter backfill. Taking into account these factors allows to increase the adequacy of the mathematical model for the studied physical processes. Numerical solutions of the corresponding nonlinear boundary value problem are found by the finite element method.

Further directions of the authors' research will concern the construction of the studied area and conducting numerical experiments to solve the obtained boundary value problem and compare the obtained results with field experiments. It is planned to use the free software environment FreeFem++ for conducting numerical experiments.

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