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Mathematical modeling of suspension filtration on a rapid filter at an unregulated rate

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Vadim Poliakov
Leading Researcher of Department of Applied Hydrodynamics
Institute of Hydromechanics NASU
Kiev, Ukraine
V.Polyakov.igm@gmail.com

***Abstract* — Mathematical modeling of detachable filtration in the split filter flow mode was carried out using exact and approximate analytical methods. An engineering technique has been developed to determine the duration of filter cycles, based on quality, economic and technical criteria. Determination of rational filter medium height was considered for two materials differed significantly by economic indexes.**

***Keywords* — filtration; modeling; suspension; exact solution; height; split filter flow.**

I. Introduction

Clarification of low-concentration aqueous suspensions is currently carried out, as a rule, on rapid filters [1] in two modes – with adjustable (constant) [2, 3] and unregulated (variable) speed. In recent decades, the second mode has been especially widely used in water purification practice [4, 5]. To increase the performance of the filters, they are equipped with special storage tanks, from which the initial suspension is fed to a layer of porous, well-absorbing material (filter medium). Typically, the suspension is fed to the filter at a constant discharge, which significantly exceeds the throughput of a clean and especially clogged medium. Therefore, the specified is discharge distributed between the tank and the filtration flow. With the progressive particles deposition in the medium layer, its hydraulic resistance increases, which accelerates the level rise in the tank. But on the other hand, the piezometric head at the inlet to the medium increases, which leads to increased suspension flow and increased output of the filter structure.

II. Results and Discussion

Mathematical modeling of the detachable filtration in the split filter flow mode was carried out using exact and approximate analytical methods. It is based on a complex mathematical model with non-linear effects and variable model coefficients. The assumed model consists of three interconnected compartments – clarification, liquid flow and hydraulic. The clarification compartment describes the transport of particles of the suspension in the medium using convective mechanism (diffusion mechanism makes an insignificant contribution) and the exchange between the solid (includes the medium elements and the already formed deposit) and liquid phases. The equations of the mass transfer and mass exchange kinetics

$$V(t)\frac{\partial C}{\partial z}+\frac{\partial S}{\partial t}=0,\quad \frac{\partial S}{\partial t}=\alpha(V)C-\beta(V)S,$$

form the basis of the first compartment. Here C and S are the volumetric concentrations of the suspended and deposited suspension particles. The key role in interfacial mass transfer is played by The adhesion of suspended particles to the solid phase plays the key role in interfacial mass transfer. It is the process that directly ensures the removal of dispersed contamination from the aqueous suspension, and its features are reflected in the linear kinetic equation. It has been established experimentally and theoretically that both mass-exchange coefficients, namely, the coefficients of suspension particles adhesion rate α and also of detachment rate β depend significantly on the filtration rate V, and the relationships between them are often almost linear. Therefore, the following approximation dependencies are valid

$$\alpha(V)=\alpha_V V^l,\qquad \beta=\beta_V V^q.$$

Here α_V, β_V, l, q are the empirical constants. Then, due to the variability V, the mass-exchange coefficients also change with time. Moreover, since the value $V(t)$ is initially unknown, the indicated coefficients turn out to be unknown functions of time. Thus, the solution of the second **compartment** and the corresponding mathematical problem as a whole are much more difficult. Due to the variability of V, the second **compartment** is closely related to the first **compartment**. It includes the equations of the laminar non-inertia motion (Darcy's law), as well as the equations characterizing the regular increase in the hydraulic resistance of the gradually clogged medium and the composition of the deposit

$$V(t)=-k(S_S)\frac{\partial h}{\partial z},$$

$$k(S_S)=k_0 f_k(S_S)=k_0\left[1-\left(\frac{S_S}{n_0}\right)^{m_1}\right]^{m_2},$$

$$S_S(S)=\gamma(S)S.$$

Here k, k_0 are the current and initial hydraulic conductivities; S_S is the volumetric concentration of the deposit; h is the piezometric head; n_0 is the porosity of the pure medium; m_1, m_2, γ are the empirical coefficients. It is important that the deposit contains predominantly bound water. The filtration process is determined, firstly, by the permeability of the bed with the deposit, and secondly, by the suspension level in the tank. The dynamics of this level is described by the equation of balance of the suspension above the fixed bed

$$\omega \frac{dH}{dt} = Q_{in} - \omega V(t);$$

$$t = 0, \quad H = H^0; \quad z = L, \quad h = H_{out} + R\omega^2 V^2(t).$$

Here ω is the area of the filter medium surface; Q_{in} – расход подаваемой на фильтр суспензии; H_{out} is the piezometric head in the filtrate collector; R is the hydraulic resistance of the filter communications. It establishes an equality between the suspension that arrives at the filter (discharge Q_{in}), retards in the tank (level $H)$ and filtered through the porous medium (rate $V)$. At the same time, due to a change in the filtration rate, the pressure losses in the inlet and outlet communications of the filter are also reduced. The corresponding equations supplement the third compartment. We use, for instance, for diverting communications

$$h(L,t) = H_{out} + R\omega^2 V^2(t).$$

For a particular, but in fact often encountered in practice, case of a linear relationship between α and V, an exact solution was obtained for the correctly posed mathematical problem of filtration in the second mode. The importance of this solution is all the more obvious because, due to the limited changes V at rapid filters, possible nonlinear relationships α with V are suitable for linear approximation. The specified solution has an integral form, a set of formulas derived from it make it possible, using standard mathematical analysis software packages, to calculate easily the spatial and temporary changes in the concentrations of suspended and deposited particles of the suspension within the medium layer and at its boundaries, the distribution of head and its general losses in the bed and the transport system of the filter structure, to control the level rise in the tank. The solution of the clarification compartment in particular case of $l = q = 1$ is expressed by the dependencies in dimensionless form for $\bar{C}$, $\bar{S}$

$$\bar{S}(\bar{z},\bar{t}) = \bar{\alpha}_V e^{-\bar{\alpha}_V \bar{z}} \int_0^{t} e^{-\bar{\beta}_V \varsigma} I_0\left(2\sqrt{\bar{\alpha}_V \bar{\beta}_V \bar{z} \varsigma}\right) d\varsigma,$$

$$\bar{C}(\bar{z},\bar{t}) = e^{-\bar{\alpha}_V \bar{z}} \left[e^{-\bar{\beta}_V \bar{t}} I_0\left(2\sqrt{\bar{\alpha}_V \bar{\beta}_V \bar{z}\bar{t}}\right) + \right.$$

$$\left. + \bar{\beta}_V \int_0^{\bar{t}} e^{-\bar{\beta}_V \varsigma} I_0\left(2\sqrt{\bar{\alpha}_V \bar{\beta}_V \bar{z} \varsigma}\right) d\varsigma \right].$$

Here the symbol “ – “ means that the corresponding value is dimensionless one; I_0 is the Bessel zero-order function of the first kind and imaginary variable.

An engineering technique has been developed to determine the duration of filter cycles (time until the next filter backwashing), based on three criteria – quality, economic and technical. The first criterion limits the filtration time due to an excessive decrease in the protective ability of the medium, the second criterion limits the filtration time due to an prohibitive decrease in filter performance, and the third criterion – due to overflow of the tank. Generally, the aim of the technological calculations was to determine the relative time parameters $\left(\bar{t}_p, \bar{t}_V, \bar{t}_H\right)$ which characterize filter efficiency coming from current contamination of the filtrate $\left(\bar{t}_p\right)$, the filter productivity $\left(\bar{t}_V\right)$, the suspension supply in the tank $\left(\bar{t}_H\right)$ and finally the duration of the filter run $\left(\bar{t}_f\right)$ as functions of the mass-exchange coefficients. Dependencies designed to predict the development of the technological process as a whole and its components, to substantiate technological and design parameters were illustrated by many examples with typical initial data. It was shown that in some cases it was advisable to continue filtration even after filling the tank. The analysis of the sensitivity of the filtration characteristics with respect to the model coefficients was carried out, which will improve the planning of experimental studies.

Determination of rational filter medium height L was of special attention. Two usual cases with medium material were consequently considered, which differed significantly by economic indexes. In the first case the specified material from local production wastes is cheap and available in unlimited amount. Then the value L doesn’t influence investments and can increase as high as the filter run becomes maximal. In the second case the material is of high price because of considerable expenses for transport, preparation and usually its amount is strictly limited. Then such material must be used economically. Subsequently, L was varied in technological analysis at constant filter medium volume.

Therefore, an effective tool has been developed for a comprehensive engineering calculation of rapid filters at an unregulated filtration rate, which allowed to predict temporal and space changes in the filtration characteristics for the reagent detachable filtration, rationally select the algorithm of the filter operation and its main constructive parameter. Then it is a reliable basis for making rational design and technological decisions.

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