MCIT
Modeling, control & information technologies

Computer modeling of differential games

https://doi.org/10.31713/MCIT.2021.03

Andrii Bardan
department of applied mathematics
Yuriy Fedkovych Chernivtsi National University
Chernivtsi, Ukraine
bardan.andrii@chnu.edu.ua

Yaroslav Bihun
department of applied mathematics
Yuriy Fedkovych Chernivtsi National University
Chernivtsi, Ukraine
y.bihun@chnu.edu.ua

***Abstract* — This paper uses differential games for viewing with simple movement and gives examples of viewing processes. Software has been developed and computer modeling of several methods of interaction in a conflict-driven environment has been introduced.**

***Keywords* — computer modeling; differential games; differential equations; persecution; game theory (key words).**

I. Introduction

Differential game theory, as a branch of control theory, studies the problems of decision-making in a conflict environment of several objects. Similar situations often arise in the economy, ecology and other areas of human life. First and foremost, the theory of differential games allows to solve a large number of such applied problems. Its formation is associated with the names of R. Isaacs [1], L.S. Pontryagina [2]. Ukrainian School of Differential Games was founded by Professor B.M. Pshenychnyy, its development thanks to the work of A.O. Chykria [3] and his students, in particular for differential-difference systems in [4].

II. Problem Formulation

Differential game theory emerged as a result of the mathematical implementation of certain technical problems. In the course of such implementation it is necessary to strive to ensure that by choosing the most important features of the technical problem, it remains available for mathematical modeling. Thus, the problem statement should not be given in complete isolation from technical problems.

To have a specific example of the situation, imagine that one plane is chasing another. The purpose of the first plane is to catch up with the second, the purpose of the second is to escape persecution. Each pilot controls his aircraft, having his own goal and using certain information about the situation. The information consists of two parts, the first is a complete knowledge of the technical capabilities of both aircrafts, the second is information about the behavior of your own aircraft and enemy aircraft. Information about the behavior of an aircraft may include various data about their condition at a certain point in time in the past, but nothing is known about the future behavior of an aircraft, because they are controlled and at any point in time the pilot can change the position of the rudder, thereby changing behavior of his own plane. In fact, each of the pilots can receive information about the enemy only with some delay, but there is no need to include this fact in the implementation. Moreover, we can even assume the known behavior of the enemy with some advance and build a mathematical idealization on this basis, and then show that the resulting theory can be used to approximate the real problem.

The implementation of a mathematical description of the persecution process is not an easy task. Two controlled objects take part in this process: a pursuer and a fugitive. The state of each of the objects at any point of time is determined by its phase vector. The phase vector of the pursuer is denoted by x, and the phase vector of the fugitive – by y, the equation of controlled objects can be written in the form:

$$\dot{x} = f(x, u), \dot{y} = g(y, v), \qquad (1)$$

where the point means the time derivative, and u and v are the essence of control, that are the parameters located on the right side of the equation. Each of the parameters belongs to a certain constraint set:

$$u \in P, \ v \in Q,$$

where P and Q are sets of arbitrary nature. If the control of u becomes a given function of time t, ie $u = u(t)$, then the first of equations (1) becomes an ordinary differential equation, which can be solved for a given initial value $x(0) = x_0$. The same applies to the second equation (1). Since x and y are phase vectors, each of them splits into two parts:

$$x = (x_1, x_2), y = (y_1, y_2),$$

where x_1 and y_1 determine the geometric position of objects, a x_2 and y_2 – their velocities.

It is believed that the process of persecution ends at the moment of equality:

$$x_1 = y_1, \qquad (2)$$

that is, when objects are geometrically matched.

Equation (1) does not describe the motion of objects, but only their possibility, because with different controls $u = u(t)$ and $v = v(t)$ different motion will occur. Therefore, in the example with aircraft, equation (1) describes the technical capabilities of aircraft.

The process of persecution itself can be viewed from two opposite points of view.

1. You can identify yourself with the persecutor. In this case, the goal will be to complete the pursuit process, then control of u is at our complete disposal to achieve this goal. Thus, at each point in time t we must construct the value $u(t)$ of the control u, knowing equation (1), ie the first part of the information, and

using its second part in the form of knowledge of the functions *x(s), y(s), v(s)* on the segment $t-\theta \leq s \leq t$, where θ — is a correspondingly selected positive number.

2. You can identify yourself with a fugitive. In this case, the goal will be to escape the pursuer, which means to fail to fulfill equality (2). Then control v is at our disposal to achieve this goal. Therefore, at each point in time t we must construct the value $v(t)$ of control v, knowing the solution of equation (1). This way, having the first part of the information, and using the second part of it in the form of values of functions *x(s), y(s), u(s)* on the segment $t - \theta \leq s \leq t$.

Such a mathematical realization of the process of persecution inevitably splits the problem into two different problems: the problem of persecution and the problem of escape.

The differential game from the process of persecution arises as a result of the natural desire to simplify the notation, namely, instead of two phase vectors x and y we introduce one vector: $z = (x, y)$, creating the phase space of the game R as the direct sum of phase spaces of both projects [3]. Then the pair of equations (1) is written as one equation:

$$\dot{z} = F(z, u, v), \quad (3)$$

and relation (2) defines in the vector space R some subset of M.

III. Computer Modeling of Processes

To demonstrate the considered theoretical results, using a modern graphical development environment Godot Engine [5], an application was created that allows you to simulate the course of a differential game with a given input data and visualize the trajectory of objects. A two-dimensional space with two different types of objects was created for visualization: the pursuer is a red sphere, and the fugitive is a blue sphere. After starting the visualization process, the objects begin to move iteratively at a given maximum speed, in the appropriate direction to the selected method. The camera follows the pursuer, but it is always possible to switch to the top view, which also shows the path taken by the objects. Images of the created application will be used further as illustrations to the given examples.

The following methods of behavior are most typical for a fugitive [4]:

- linear motion;
- movement at right angles;
- movement from the pursuer.

In turn, the following methods of behavior are most typical for the persecutor:

- running curve method;
- constant bias angle method;
- method of parallel persecution.

Most of these behaviors are based on the complete information that each object has about all other objects in the system. Namely, information about the current position of the object, and the history of its movement.

Consider in more detail the methods of movement of the pursuer:

1. The running curve method is the simplest guidance method. Characteristic of this method is that the direction vector of the pursuer at each point in time exactly coincides with the direction to the target, and the bias angle equals zero.

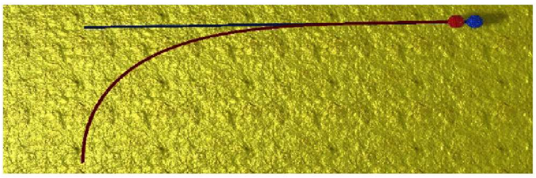

Figure 1. The trajectory of movement by the method of running curve

Fig. 1 shows a partial case of a running curve. If the trajectory of the fugitive is a straight line, then the trajectory of the pursuer is a Euler spiral. The general type of control can be described as follows:

$$u_i = \frac{x_i - y_i}{|x_i - y_i|}\sigma.$$

The running curve method has one significant drawback - a fairly long end time of the game, but it is a very popular method because of its simplicity and guaranteed results, provided that the speed of the fugitive is less than the speed of the pursuer.

2. The constant bias angle method is a modification of the running curve method. The vector of the pursuer's direction at each moment of time is constructed by means of a vector of a direction on the purpose with turn on the angle of bias set by the user. This angle corrects the pursuer’s trajectory (see Fig. 2). With the correct value of the angle, the method has a better completion time than the running curve method, but it does not always guarantee its end. The use of this method is impractical if the trajectory of the pursuer is unpredictable.

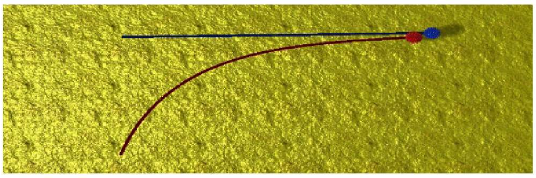

Figure 2. The trajectory of motion by the method of a constant angle of bias

3. The method of parallel pursuit is a modification of the method of constant angle of bias. In the case of parallel pursuit, the bias angle K_P is calculated, depending on the current position of the fugitive and the direction of his movement, and is different at each step [1]. The method is often used in practice to guide rocketry.

According to Isaacs [1], the angle of bias K_P, can be found in this equality:

$$\sin(\psi_e) = -n \times \sin(\psi_p), \text{ де } n = \frac{Ve}{Vp}.$$

If the target moves evenly and rectilinearly, the pursuer will move on the most effective trajectory - in a straight line, then the end time of the game will be minimal and in this case this method is the most effective of the above (see Fig. 3). The method of parallel pursuit guarantees the end of the game.

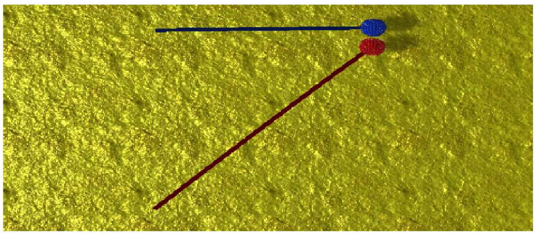

Figure 3. The trajectory of the method of parallel pursuit

Consider in more detail the methods of movement of the fugitive:

1. The method of linear motion is a normal motion in a straight line, in which the object does not change its direction, it moves in a constant static direction. This movement can reflect the fugitive's desire to reach a certain goal at any cost as quickly as possible. For example, a fugitive intends to reach an area that is inaccessible to the pursuer. However, such behavior is the easiest for the persecutor, because it is completely predictable.

2. The method of movement at right angles is a movement with a constant right angle between the direction of movement of the fugitive and the pursuer.

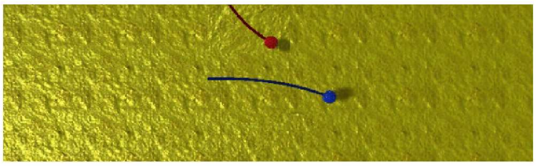

Figure 4. The trajectory of motion at right angle

3. Method of movement from the pursuer - is to choose the movement in the opposite direction, to the current position of the pursuer. For instance, if the pursuer is in the direction (-1.0) from the fugitive, the direction of movement of the fugitive will be equivalent to the vector (1.0) (see Fig. 5). The purpose of such a movement is to be as far away from the pursuer as possible. This behavior will have the best time characteristics for the fugitive.

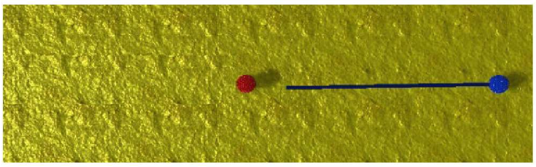

Figure 5. The trajectory of movement from the pursuer

There are more complex modifications of differential games, when there is a group of persecutors, or there is a group of fugitives. The goal of a group of persecutors is to catch all fugitives, and the goal of fugitives is that at least one has to escape from persecution. It is believed that the fugitive will be caught if the pursuers form a convex polygon around him. Thus, the task of the pursuers is to form this convex polygon, for which they move by the method of parallel pursuit. The algorithm of the fugitive's motion depends on the initial conditions (it may belong to the inner part of the convex hull). In some cases, the success of one of the fugitives is guaranteed. For example, if there is a task where we have 4 pursuers and 2 fugitives, under any initial conditions and positions of players, at least one of the fugitives can avoid being caught [4].

IV. CONCLUSIONS

Thus, in this paper were considered: the problem statement of the theory of differential games, mathematical models of conflict-controlled processes, examples of simple persecution processes. Moreover, the main methods of player management were described. The advantages and disadvantages of each of the methods are investigated. Graphical illustrations of methods are given, by means of the specially created application which allows to model a course of differential game with the set input data. The developed software also allows to model complex modifications of differential games - group pursuits.

REFERENCES

[1] R. Isaacs, “Differential Games: A Mathematical Theory with Applications to Warfare and Pursuit, Control and Optimization,” Dover Publications, 1999.

[2] L.S. Pontryagin, “Selected scientific works,” vol. 2, Nauka, 1988, pp. 576–579.

[3] A.I. Chikriy, “Conflict-controlled processes,” Kyiv: Naukova dumka, 1992, pp. 383–385.

[4] E.A. Lyubarschuk, “Linear nonstationary differential-difference convergence games,” Yuri Fedkovych Chernivtsi National University, 2017.

[5] J. Linietsky, A. Manzur, and the Godot community (CC-BY 3.0), “Godot's documentation,” electronic resource, access mode: URL: https://docs.godotengine.org/en/, 2014–2020.