Computer prediction of technological modes of rapid cone-shaped adsorption filters with automated discharge of part of heat from separation surfaces in filtering mode

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***Abstract* — In the paper a mathematical model of technological modes of filtration with automated removal of part of heat from interface surfaces (water purification from multicomponent impurities), backwashing, chemical regeneration and direct washing of rapid cone-shaped adsorption filters with chemical regeneration of piecewise homogeneous porous loads while maintaining constant velocities of the respective modes is formulated. The proposed models in the complex allow to conduct computer experiments to investigate the change in the concentrations of components of a multicomponent impurity in the filtration stream and on the surface of the loading adsorbent, retained by both physical and chemical adsorption, filtration flow temperature, filtration coefficient, active porosity and pressure along the filter height and on their basis to predict more optimal options for the use of adsorbents of each loading layer and increase the protective time of rapid cone-shaped adsorption filters with automated heat removal from the interface surfaces in filter mode.**

***Keywords* — mathematical model; computer prediction; filtration; washing; chemical regeneration; impurity; adsorption; desorption; temperature; rapid cone-shaped filter; piecewise homogeneous porous loading.**

I. Introduction

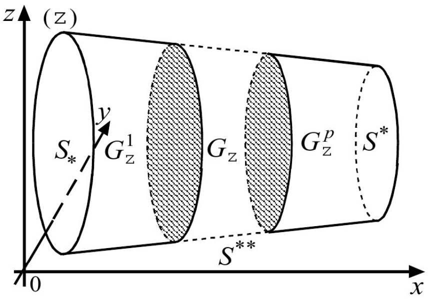
For adsorption of large volumes of water, rapid adsorption filters with one- and multilayer porous regenerative loads are increasingly used while maintaining a constant filtration rate. With the help of regeneration (using special chemicals) the adsorption properties of most filter materials are almost completely restored [1–3].

The ever-increasing demand for purified water in the economy and the growing cost of filter materials require research on the one hand their more optimal use and increase the duration of rapid adsorption filters in the filtration mode by choosing their shape, layer height, in particular, taking into account the effects of temperature on the internal kinetics of mass transfer, and on the other is the regeneration of porous loads for their reuse.

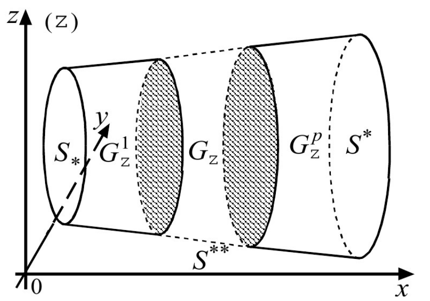
Filtration of water through porous loads of rapid adsorption filters is a special case of the movement of liquids through porous materials and the laws of this movement are fully covered in the works of D. M. Mintz, L. S. Leibenzon, N. N. Pavlovsky, S.A. Schubert and others researchers [4–8]. It is established that the linear Darcy's law is usually valid when filtering water through porous materials, ie the laminar regime of water motion takes place. The speed of the adsorption process depends on the concentration, nature and structure of the components of the multicomponent impurity, filtration rate, filtration flow temperature, type and properties of the adsorbent [8]. As mathematical models for predicting the processes of "filtering-regeneration" of porous loads, domestic researchers often use the model of DM Mintz [6, 7] with constant speeds of the respective processes and temperature or some modification (advanced model). In [9, 10] its corresponding spatial models for prediction of technological modes of filtration, backwashing, chemical regeneration and direct washing of rapid cone-shaped adsorption filters with piecewise homogeneous porous loadings taking into account influence of temperature effects on internal kinetics of mass transfer are offered. These models in combination by taking into account the influence of temperature and filtration flow rate along the filter height on the coefficients that characterize the rates of mass transfer during physical and chemical adsorption and desorption, filtration coefficient, allow computer experiments to predict more optimal use of adsorbents the loading layer and the time intervals of the filters, respectively, in the modes of filtration, backwashing, chemical regeneration and direct washing with constant speeds of the respective modes. An urgent task is to generalize the relevant models in the case of computer forecasting of technological modes of rapid cone-shaped adsorption filters with automated removal of part of the heat from the interface surfaces in the filtration mode.

II. MATHEMATICAL MODEL

We will form spatial boundary value problems for forecasting the main technological modes of operation of rapid cone-shaped adsorption filters with chemical regeneration of piecewise homogeneous porous loads for the model domain $G = G_z \times (0,\infty)$, G_z is a spatial one-connected domain ($z = (x, y, z)$), bounded by given smooth, orthogonal to each other along the edges, two equipotential surfaces S_*, S^* and one flow surface S^{**} and divided into p subdomains G_z^r ($r = \overline{1, p}$) by some given $p-1$ equipotential surfaces S_{*r}^* ($r = \overline{1, p-1}$) (fig. 1).



a



b

Figure. 1. Spatial filtering domains G_z for model problems of forecasting technological modes: a is filtration and direct washing; b is backwash and chemical regeneration

Considering that in the filtration mode with automated heat removal from the interface surfaces the convective components of heat and mass transfer and physical and chemical adsorption of impurities prevail over the contribution of diffusion and physical desorption, in the chemical regeneration mode the convective components of heat and mass transfer and physical and chemical over the contribution of diffusion and physical adsorption, and in the modes of reverse and direct washing convective components of heat and mass transfer and physical desorption and chemical adsorption prevail over the contribution of diffusion and physical adsorption, spatial model problems for forecasting technological modes of operation of rapid cone-shaped adsorption filter at constant speeds of corresponding modes taking into account the reverse effect of process characteristics (concentrations of multicomponent impurity components in filtration flow and loading adsorbent surface, filtration flow temperature) on loading characteristics active porosity, the coefficients that characterize the rates of mass transfer during physical and chemical adsorption and desorption of components of a multicomponent impurity) will consist of the equations of motion of the filtration flow according to Darcy's law, equations to determine the change in the concentrations of the components of the multicomponent impurity in the filtration stream and on the surface of the loading adsorbent, held by physical and chemical adsorption, filtration flow temperature, filtration coefficient, active porosity and pressure along the filter height respectively for filtration modes:

$$\begin{cases} (\sigma \cdot C)'_t = div\,(D \cdot grad\ C) - \vec{v} \cdot grad\ C - \\ -\alpha \cdot C + \beta \cdot U - \alpha^* \cdot C, \\ (\sigma \cdot U)'_t = div\,(D^* \cdot grad\ U) + \alpha \cdot C - \beta \cdot U, \\ (\sigma \cdot W)'_t = div\,(D^{**} \cdot grad\ W) + \alpha^* \cdot C, \\ (\sigma \cdot T)'_t = div\,(D^{***} \cdot grad\ T) - \vec{v} \cdot grad\ T + \\ +\gamma \cdot (\alpha \cdot C - \beta \cdot U) + \gamma^* \cdot \alpha^* \cdot C, \\ \vec{v} = -\kappa \cdot grad\ h, \\ \kappa'_t = -\mu \cdot U - \mu^* \cdot W, \\ \sigma'_t = -\lambda \cdot U - \lambda^* \cdot W, (x, y, z) \in G_z^r, r = \overline{1, p}, \end{cases}$$

chemical regeneration:

$$\begin{cases} (\sigma \cdot C)'_t = div\,(D \cdot grad\ C) - \vec{v} \cdot grad\ C + \\ +\beta \cdot U - \alpha \cdot C + \beta^* \cdot W, \\ (\sigma \cdot U)'_t = div\,(D^* \cdot grad\ U) - \beta \cdot U + \alpha \cdot C, \\ (\sigma \cdot W)'_t = div\,(D^{**} \cdot grad\ W) - \beta^* \cdot W, \\ (\sigma \cdot T)'_t = div\,(D^{***} \cdot grad\ T) - \vec{v} \cdot grad\ T + \\ +\gamma \cdot (\beta \cdot U - \alpha \cdot C) + \gamma^* \cdot \beta^* \cdot W, \\ \vec{v} = -\kappa \cdot grad\ h, \\ \kappa'_t = \mu \cdot U + \mu^* \cdot W, \\ \sigma'_t = \lambda \cdot U + \lambda^* \cdot W, (x, y, z) \in G_z^r, r = \overline{1, p}, \end{cases}$$

reverse and direct flushing:

$$\begin{cases} (\sigma \cdot C)'_t = div\,(D \cdot grad\ C) - \vec{v} \cdot grad\ C + \\ +\beta \cdot U - \alpha \cdot C - \alpha^* \cdot C, \\ (\sigma \cdot U)'_t = div\,(D^* \cdot grad\ U) - \beta \cdot U + \alpha \cdot C, \\ (\sigma \cdot W)'_t = div\,(D^{**} \cdot grad\ W) + \alpha^* \cdot C, \\ (\sigma \cdot T)'_t = div\,(D^{***} \cdot grad\ T) - \vec{v} \cdot grad\ T + \\ +\gamma \cdot (\beta \cdot U - \alpha \cdot C) + \gamma^* \cdot \alpha^* \cdot C, \\ \vec{v} = -\kappa \cdot grad\ h, \\ \kappa'_t = \mu \cdot U - \mu^* \cdot W, \\ \sigma'_t = \lambda \cdot U - \lambda^* \cdot W, (x, y, z) \in G_z^r, r = \overline{1, p}, \end{cases}$$

which are supplemented by the following boundary conditions:

$$\begin{cases} C\big|_{S_*} = c_*^*,\, C'_{\vec n}\big|_{S^*} = 0,\, C'_{\vec n}\big|_{S^{**}} = 0, \\ U\big|_{S_*} = u_*^*,\, U'_{\vec n}\big|_{S^*} = 0,\, U'_{\vec n}\big|_{S^{**}} = 0, \\ W\big|_{S_*} = u_*^*,\, W'_{\vec n}\big|_{S^*} = 0,\, W'_{\vec n}\big|_{S^{**}} = 0, \\ T\big|_{S_*} = T_*^*,\, T'_{\vec n}\big|_{S^*} = 0,\, T'_{\vec n}\big|_{S^{**}} = 0, \end{cases}$$

initial conditions:

$$\begin{cases} C\big|_{t=0} = c_0^0,\, U\big|_{t=0} = u_0^0,\, W\big|_{t=0} = w_0^0, \\ T\big|_{t=0} = T_0^0,\, h\big|_{t=0} = h_0^0,\, \kappa\big|_{t=0} = \kappa_0^0,\, \sigma\big|_{t=0} = \sigma_0^0, \end{cases}$$

and conditions of consistency on the surfaces of the section S_{*r}^* $(r=\overline{1,p-1})$:

$$\begin{cases} C\big|_{S_{r-}} = C\big|_{S_{r+}},\ D_r\cdot C'_{\vec n} - v_{r\,n}^0\cdot C\big|_{S_{r-}} = \\ = D_{r+1}\cdot C'_{\vec n} - v_{r\,n}^0\cdot C\big|_{S_{r+}}, \\ U\big|_{S_{r-}} = U\big|_{S_{r+}},\ D_r^*\cdot U'_{\vec n}\big|_{S_{r-}} = D_{r+1}^*\cdot U'_{\vec n}\big|_{S_{r+}}, \\ W\big|_{S_{r-}} = W\big|_{S_{r+}},\ D_r^{**}\cdot W'_{\vec n}\big|_{S_{r-}} = D_{r+1}^{**}\cdot W'_{\vec n}\big|_{S_{r+}}, \\ T\big|_{S_{r-}} = T - \theta\cdot T\big|_{S_{r+}},\ D_r^{***}\cdot T'_{\vec n} - v_{r\,n}^0\cdot T\big|_{S_{r-}} = \\ = D_{r+1}^{***}\cdot T'_{\vec n} - v_{r\,n}^0\cdot T - \theta\cdot T\big|_{S_{r+}}, \\ h\big|_{S_{r-}} = h\big|_{S_{r+}},\ \kappa_r\cdot h'_{\vec n}\big|_{S_{r-}} = \kappa_r\cdot h'_{\vec n}\big|_{S_{r+}}, \\ D_r\cdot C'_{\vec n} - v_{r\,n}\cdot C + D_r^*\cdot U' + D_r^{**}\cdot W'_{\vec n}\big|_{S_{r-}} = \\ = D_{r+1}\cdot C'_{\vec n} - v_{r\,n}\cdot C + D_{r+1}^*\cdot U'_{\vec n} + D_{r+1}^{**}\cdot W'_{\vec n}\big|_{S_{r+}}, \\ (\sigma\cdot(C+U+W))'_t\big|_{S_{r-}} = (\sigma\cdot(C+U+W))'_t\big|_{S_{r+}},\ r=\overline{1,p-1}, \end{cases}$$

where $C=C(x,y,z,t)$, $U=U(x,y,z,t)$ i $W=W(x,y,z,t)$ is the concentration of impurities in the filtration stream and on the surface of the loading adsorbent, respectively, retained by physical and chemical adsorption, $T=T(x,y,z,t)$ is filtration flow temperature, $h=h(x,y,z,t)$ is pressure, $\kappa=\kappa(x,y,z,t)$ is filtration coefficient, $\sigma=\sigma(x,y,z,t)$ is active porosity, $\vec v=\vec v(v_x,v_y,v_z)$ is filtration rate vector, $\nu=|\vec v|=\sqrt{v_x^2(x,y,z)+v_y^2(x,y,z)+v_z^2(x,y,z)}>>0$, $\vec n$ is external normal to the corresponding surface, D, D^* i D^{**} is diffusion coefficients of impurities in the filtration stream and on the surface of the loading adsorbent, respectively, retained by physical and chemical adsorption, $D=\{D_r,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $D_r=\varepsilon\cdot d_{r,0}$, $d_{r,0}>0$ $(r=\overline{1,p})$, $D^*=\{D_r^*,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $D_r^*=\varepsilon\cdot d_{r,0}^*$, $d_{r,0}^*>0$ $(r=\overline{1,p})$, $D^{**}=\{D_r^{**},(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $D_r^{**}=\varepsilon\cdot d_{r,0}^{**}$, $d_{r,0}^{**}>0$ $(r=\overline{1,p})$, D^{***} is coefficient of thermal conductivity, $D^{***}=\{D_r^{***},(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $D_r^{***}=\varepsilon\cdot d_{r,0}^{***}$, $d_{r,0}^{***}>0$ $(r=\overline{1,p})$, α i β is coefficients characterizing the mass transfer rate, respectively, during physical adsorption and desorption of impurities, for the model problem of predicting the filtration mode $\alpha=\{\alpha_r,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\alpha_r=\sum_{s_1=0}^{2}\sum_{s_2=0}^{2-s_1}\varepsilon^{s_1+s_2}\cdot\alpha_{r,s_1,s_2}\cdot v^{s_1}\cdot T^{s_2}$, $\alpha_{r,s_1,s_2}\in R$ $(r=\overline{1,p}$, $s_1,s_2=\overline{0,2})$, $\beta=\{\beta_r,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\beta_r=\varepsilon\cdot\sum_{s_1=0}^{2}\sum_{s_2=0}^{2-s_1}\varepsilon^{s_1+s_2}\cdot\beta_{r,s_1,s_2}\cdot v^{s_1}\cdot T^{s_2}$, $\beta_{r,s_1,s_2}\in R$ $(r=\overline{1,p}$, $s_1,s_2=\overline{0,2})$, and model problems of forecasting the modes of chemical regeneration, reverse and direct washing $\alpha=\{\alpha_r,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\alpha_r=\varepsilon\cdot\sum_{s_1=0}^{2}\sum_{s_2=0}^{2-s_1}\varepsilon^{s_1+s_2}\cdot\alpha_{r,s_1,s_2}\cdot v^{s_1}\cdot T^{s_2}$, $\alpha_{r,s_1,s_2}\in R$ $(r=\overline{1,p}$, $s_1,s_2=\overline{0,2})$, $\beta=\{\beta_r,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\beta_r=\sum_{s_1=0}^{2}\sum_{s_2=0}^{2-s_1}\varepsilon^{s_1+s_2}\cdot\beta_{r,s_1,s_2}\cdot v^{s_1}\cdot T^{s_2}$, $\beta_{r,s_1,s_2}\in R$ $(r=\overline{1,p}$, $s_1,s_2=\overline{0,2})$, α^* is coefficient that characterizes the rate of mass transfer during chemical adsorption of impurities, for model problems of predicting filtration modes, reverse and direct washing, $\alpha^*=\{\alpha_r^*,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\alpha_r^*=\sum_{s_1=0}^{2}\sum_{s_2=0}^{2-s_1}\varepsilon^{s_1+s_2}\cdot\alpha_{r,s_1,s_2}^*\cdot v^{s_1}\cdot T^{s_2}$, $\alpha_{r,s_1,s_2}^*\in R$ $(r=\overline{1,p}$, $s_1,s_2=\overline{0,2})$, β^* is coefficient that characterizes the rate of mass transfer during chemical desorption of impurities, for the model problem of predicting the regime of chemical regeneration, $\beta^*=\{\beta_r^*,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\beta_r^*=\sum_{s_1=0}^{2}\sum_{s_2=0}^{2-s_1}\varepsilon^{s_1+s_2}\cdot\beta_{r,s_1,s_2}^*\cdot v^{s_1}\cdot T^{s_2}$, $\beta_{r,s_1,s_2}^*\in R$ $(j=\overline{1,m}$, $r=\overline{1,p}$, $s_1,s_2=\overline{0,2})$, γ is coefficient characterizing the rate of change of the filtration flow temperature during physical adsorption and desorption of impurities, $\gamma=\{\gamma_r,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\gamma_r\in R$ $(r=\overline{1,p})$, γ^* is coefficient characterizing the rate of change of the filtration flow temperature during chemical adsorption and desorption of impurities, $\gamma^*=\{\gamma_r^*,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\gamma_r^*\in R$ $(r=\overline{1,p})$, μ i λ is coefficients characterizing the rate of change, respectively, of the filtration coefficient and the active porosity of the load during physical adsorption and desorption of impurities, $\mu=\{\mu_r,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\mu_r=\varepsilon\cdot\sum_{s=0}^{2}\varepsilon^s\cdot\mu_{r,s}\cdot T^s$, $\mu_{r,s}\in R$ $(r=\overline{1,p}$, $s=\overline{0,2})$, $\lambda=\{\lambda_r,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\lambda_r=\varepsilon\cdot\bar\lambda_r$ $(r=\overline{1,p})$, μ^* i λ^* is coefficients characterizing the rate of change, respectively, of the filtration coefficient and the active porosity of the load during chemical adsorption and desorption of impurities, $\mu^*=\{\mu_r^*,(x,y,z)\in G_z^r, r=\overline{1,p}\}$, $\mu_r^*=\varepsilon\cdot\sum_{s=0}^{2}\varepsilon^s\cdot\mu_{r,s}^*\cdot T^s$,

$\mu^*_{r,s} \in R$ ($r=\overline{1,p}$, $s=\overline{0,2}$), $\lambda^* = \{\lambda^*_r, (x,y,z)\in G^r_z, r=\overline{1,p}\}$, $\lambda^*_r = \varepsilon\cdot\bar{\lambda}^*_r$ ($r=\overline{1,p}$), θ_r is coefficient that for the model problem of forecasting the filtration mode characterizes the rate of temperature change at the interface S^*_{*r} due to the automated removal of part of the heat, $\theta_r = \begin{cases} 0, \ T \le T^0_r, \\ \theta^*_r, T \ge T^*_r, \end{cases}$ $\theta^*_r \in R$, $T^0_r \in R$, $T^*_r \in R$ ($r=\overline{1,p-1}$), $\alpha_r = \alpha_r(x,y,z,t)$, $\beta_r = \beta_r(x,y,z,t)$, $\alpha^*_r = \alpha^*_r(x,y,z,t)$, $\beta^*_r = \beta^*_r(x,y,z,t)$, $\mu_r = \mu_r(x,y,z,t)$, $\bar{\lambda}_r = \bar{\lambda}_r(x,y,z,t)$, $\mu^*_r = \mu^*_r(x,y,z,t)$, $\bar{\lambda}^*_r = \bar{\lambda}^*_r(x,y,z,t)$ ($r=\overline{1,p}$) is continuous limited functions, ε is small parameter ($\varepsilon > 0$), $c^0_0 = \{c^0_r, (x,y,z)\in G^r_z, r=\overline{1,p}\}$, $u^0_0 = \{u^0_r, (x,y,z)\in G^r_z, r=\overline{1,p}\}$, $w^0_0 = \{w^0_r, (x,y,z)\in G^r_z, r=\overline{1,p}\}$, $T^0_0 = \{T^0_r, (x,y,z)\in G^r_z, r=\overline{1,p}\}$, $h^0_0 = \{h^0_r, (x,y,z)\in G^r_z, r=\overline{1,p}\}$ $\kappa^0_0 = \{\kappa^0_r, (x,y,z)\in G^r_z, r=\overline{1,p}\}$, $\sigma^0_0 = \{\sigma^0_r, (x,y,z)\in G^r_z, r=\overline{1,p}\}$, $c^*_* = c^*_*(M,t)$, $c^0_r = c^0_r(x,y,z)$, $u^*_* = u^*_*(M,t)$, $u^0_r = u^0_r(x,y,z)$, $w^*_* = w^*_*(M,t)$, $w^0_r = w^0_r(x,y,z)$, $T^*_* = T^*_*(M,t)$, $T^0_r = T^0_r(x,y,z)$, $h^0_r = h^0_r(x,y,z)$, $\kappa^0_r = \kappa^0_r(x,y,z)$, $\sigma^0_r = \sigma^0_r(x,y,z)$ ($r=\overline{1,p}$) is quite smooth functions, consistent with each other on the edges of the domain G [11], $M \in S_*$, $v^0_{r\,n}$ і $v_{r\,n}$ ($r=\overline{1,p-1}$) is respectively, the initial and current normal velocity components on the interface S^*_{*r} ($r=\overline{1,p-1}$).

III. PROBLEM SOLVING AND CONCLUSIONS

Algorithms for numerical-asymptotic approximations of solutions of the corresponding nonlinear singularly perturbed boundary value problems for model regions of conical shape bounded by two equipotential surfaces and the flow surface are obtained analogously to [9–11].

The proposed models for predicting the processes of “filtration-regeneration” of rapid cone-shaped adsorption filters with piecewise homogeneous porous loads in the complex by taking into account the influence of temperature effects on the internal kinetics of mass transfer similarly [9, 10] allow computer experiments to better investigate the concentrations of the components of the multicomponent impurity in the filtration flow and on the surface of the loading adsorbent, due to physical and chemical adsorption, filtration flow temperature, filtration coefficient and active porosity in each loading layer along the filter height on their basis to predict more optimal use of adsorbents of each loading layer and time intervals of filters, respectively, in the modes of filtration, backwashing, chemical regeneration and direct washing with constant speeds of the respective modes.

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