Minimizing the time for UAV to reach a moving target

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Elkhan N.Sabziev
Institute of Control System,
National Defense University
Baku, Azerbaijan
elkhan.sabziev@gmail.com

Lamiya N.Nabadova
Institute of Control System
Baku, Azerbaijan
nabadovalamiya@gmail.com

Abstract—**In this study, we address the control challenges of directing a flying object, specifically a drone, to a designated target, utilizing coordinates relayed from a monitoring device. Monitoring device delivers information about coordinates of suspicious object to drone. Upon receiving signals from a monitoring device about potential threats on the ground, the drone is tasked to promptly respond, identify, and neutralize the suspicious object. This article deals with the issue of controlling the process of flying snow to the target in the shortest possible time.**

Keywords—drone applications; optimal control; UAV controlling; path planning; navigation: UAV mission planning

I. INTRODUCTION

In recent years, unmanned aerial vehicles (UAVs), commonly referred to as drones, have gained substantial attention in various sectors including defense, logistics, agriculture, entertainment, and surveillance. One of the principal challenges in the operation of drones is the achievement of optimal control, ensuring they reach their desired destinations swiftly and efficiently. Time is often a critical factor, especially in applications like emergency deliveries or quick response missions. Efficient flight not only ensures timely completion of a task but also aids in energy conservation, maximizing the operational period of the drone on a single charge.

In recent years, unmanned aerial vehicles (UAVs), commonly referred to as drones, have gained substantial attention in various sectors including defense, logistics, agriculture, entertainment, and surveillance [1, 2]. This research proposes algorithm to control the drone when it must reach the target during shortest time to complete any task. İn this case, it is considered that if monitoring system detects suspicious object on the ground of the border zone and delivers information about location of the object to drone, drone should get the intended target and annihilate it. For this situation drone should reach the detected object as fast as possible. So, it must be controlled optimally taking into account its physical limitations. Using the suggested model, we compute the drone's control function values to ensure it reaches the identified target in the shortest possible time and accomplishes the mission.

II. PROBLEM STATEMENT

This scientific paper is related to solving controlling process of flying drone to reach target in the shortest period of time. During arial surveillance the monitoring device the area detects a suspicious object moving in a certain direction at a certain speed. The geographical scale of the issue is such that the area can be considered a flat part. The monitoring device can determine the trajectory of the object. It is considered that this object is identified and determined that it should be neutralized. The other aircraft provided with neutralization device is informed to fly to the suspicious object and annihilate it. It is considered that the drone receiving the flight command must reach the suspicious object in the shortest possible time after rising to a certain height. Since the UAV is flying parallel to the ground since the flight altitude does not change, we will make the flight plane the same as the Earth plane.

To describe the mutual position of the UAV and the suspect object, let us introduce a rectangular OXY coordinate system with respect to the ground. Let's mark the time of the reaching process with $t \geq 0$ and the coordinates of the UAV as a function of time with $x(t) = (x_1(t), x_2(t))$. In a simple case, the governing equations of the UAV can be written as follows:

$$\begin{cases} x_1''(t) = u_1(t), \\ x_2''(t) = u_2(t). \end{cases} \tag{1}$$

Here is the $u(t) = (u_1(t), u(t))$. control function, which physically represents the ratio (momentum) of the propulsion force generated by the drone's engines to its mass. As a rule, the controllability of the aircraft is limited, which means that there is a known $u_0 > 0$ quantity determined by the power of the UAV engine that,

$$u_1^2(t) + u_t^2(t) \leq u_0 \tag{2}$$

For the sake of simplicity, we can assume that the UAV is located at the origin of coordinates at the moment $t = 0$, and let us denote its speed

$V(0) = (v_{x,1}, v_{x,2})$ at that moment. These conditions can be written as follows:

$$\begin{cases} x_1(0) = 0, \\ x_2(0) = 0, \end{cases} \quad (3)$$

$$\begin{cases} x_1'(0) = v_{x,1}, \\ x_2'(0) = v_{x,2}. \end{cases} \quad (4)$$

As mentioned above, the movement trajectory of the suspicious object is determined by monitoring device. We will consider that the movement of this object has the character of uniform linear motion. If we mark the coordinates of the suspicious object as a function of time, its trajectory can be expressed as follows:

$$\begin{cases} x_{1,n} = a_{x,1} + b_{x,1}t, \\ x_{2,n} = a_{x,2} + b_{x,2}t. \end{cases} \quad (5)$$

Here $a_{x,1}, b_{x,1}, a_{x,2}, b_{x,2}$ coefficients are known quantites. In order to track the detected object, it is necessary to choose control $u(t) = (u_1(t), u(t))$ in such a way that regardless of where the aircraft is at the moment of detection, it will change its trajectory and reach the suspicious object and begin to move along with it (papallel) and follow it. This can happen when the speed of the aircraft is greater than the speed of the detected object, i.e.

$$\sqrt{b_{x,1}{}^2 + b_{x,2}{}^2} < \sqrt{2}|u(t)| \le \sqrt{2}u_0$$

Let's mark the moment when the UAV will be controlled and reach the suspicious object with T. As the case may be, the UAV should be managed in such a way that, at the moment T its coordinates and velocity should coincide with the current coordinates and velocity of the suspect object, in other words, the following equations should be satisfied.

$$\begin{cases} x_1(T) = a_{x,1} + b_{x,1}T, \\ x_2(T) = a_{x,2} + b_{x,2}T. \end{cases} \quad (6)$$

$$\begin{cases} x_1'(T) = b_{x,1}, \\ x_2'(T) = b_{x,2}\,. \end{cases} \quad (7)$$

It is necessary to find a control $u(t)$ function that satisfies the inequality (2) so that the solution of the system of equations (1) satisfies the conditions (3) – (4), (6) – (7) and T is minimal.

III. PROBLEM SOLUTION

In order to apply the mathematical apparatus of optimal control theory with phase constraints, let us write the system (1) as a system of first order ordinary differential equations. If we substitute $x_1'(t) = x_3(t)$, $x_2'(t) = x_4(t)$, problem (1) – (7) will be as follows:

$$\begin{cases} x_1'(t) = x_3(t), \\ x_2'(t) = x_4(t), \\ x_3'(t) = u_1(t), \\ x_4'(t) = u_2(t). \end{cases} \quad (8)$$

$$\begin{cases} x_1(0) = 0, \\ x_2(0) = 0, \\ x_3(0) = v_{x,1}, \\ x_4(0) = v_{x,2.} \end{cases} \quad (9)$$

$$\begin{cases} x_1(T) = a_{x,1} + b_{x,1}T, \\ x_2(T) = a_{x,2} + b_{x,2}T. \\ \begin{cases} x_3(T) = b_{x,1}, \\ x_4(T) = b_{x,2}\,. \end{cases} \end{cases} \quad (10)$$

It is necessary to find such a $u(t)$ control function that ensures the solution of the problem (8) – (10). Ininve research [3] it is investigated that, to solve the problem it is sufficient that to chage the values of $u_1(t), u_2(t)$ control functions only once at the same time moment $\tau \subseteq (0,T)$. In other words, there is a $\tau \subseteq (0,T)$ point that flying control u changes at the $t = \tau$ moment when bringing flying object from state (8) to state (9).

In $(0,\tau)$ and (τ,T) intervals, lets denote the values of control vectors $(u_{x,1,0}, u_{x,2,0})$ and $(u_{x,1,T}, u_{x,2,T})$ accordingly. From the management theory of extremal problems, it is known that the solution of the optimization problem within the constraint (2) is realized in the case of equality, in other words, for $u_{x,1,0}, u_{x,2,0}, u_{x,1,T}, u_{x,2,T}$ quantities

$$\begin{cases} u_{x1,0}{}^2 + u_{x2,0}{}^2 = u_0^2, \\ u_{x1,T}{}^2 + u_{x2,T}{}^2 = u_0^2. \end{cases} \quad (11)$$

We can write the general solution of system (8) as follows:

$$\left.\begin{array}{l} x_1(t) = \begin{cases} \frac{1}{2}u_{x1,0}t^2 + v_{x,1}t, & 0 < t < \tau, \\ \frac{1}{2}u_{x1,T}t^2 + c_{x,1}t + e_{x,1}, & \tau < t < T, \end{cases} \\ x_2(t) = \begin{cases} \frac{1}{2}u_{x2,0}t^2 + v_{x,2}t, & 0 < t < \tau, \\ \frac{1}{2}u_{x2,T}t^2 + c_{x,2}t + e_{x,2}, & \tau < t < T \end{cases} \\ x_3(t) = \begin{cases} u_{x1,0}t + v_{x,1}, & 0 < t < \tau, \\ u_{x1,T}t + c_{x,1}, & \tau < t < T, \end{cases} \\ x_4(t) = \begin{cases} u_{x2,0}t + v_{x,2}, & 0 < t < \tau, \\ u_{x2,T}t + c_{x,2}, & \tau < t < T. \end{cases} \end{array}\right\}$$

Here $c_{x,1}, c_{x,1}, e_{x,1}, e_{x,1}$ are constant quantities. Taking account conditions (10),

$$\begin{cases} \frac{1}{2}u_{x1,T}T^2 + c_{x,1}T + e_{x,1} = a_{x,1} + b_{x,1}T, \\ \frac{1}{2}u_{x2,T}T^2 + c_{x,2}T + e_{x,2} = a_{x,2} + b_{x,2}T, \\ u_{x1,T}T + c_{x,1} = b_{x,1}, \\ u_{x2,T}T + c_{x,2} = b_{x,2}. \end{cases} \quad (12)$$

On the other hand, from continuity condition of (x_1, x_2, x_3, x_4) functions at $t = \tau$ moment:

$$\begin{cases} \frac{1}{2}u_{x1,T}\tau^2 + c_{x,1}\tau + e_{x,1} = \frac{1}{2}u_{x1,0}\tau^2 + v_{x,1}\tau, \\ \frac{1}{2}u_{x2,T}\tau^2 + c_{x,2}\tau + e_{x,2} = \frac{1}{2}u_{x2,0}\tau^2 + v_{x,2}\tau, \\ u_{x1,0}\tau + v_{x,1} = u_{x1,T}\tau + c_{x,1}, \\ u_{x2,0}\tau + v_{x,2} = u_{x2,T}\tau + c_{x,2}. \end{cases} \quad (13)$$

If we eliminate variables from $c_{x,1}, c_{x,2}, e_{x,1}, e_{x,2}$ (12), (13) systems, by taking linear combinations of different rows, we get:

$$\begin{cases} u_{x1,T}(\tau^2 - T^2) - u_{x1,0}\tau^2 = 2a_{x,1}, \\ u_{x2,T}(\tau^2 - T^2) - u_{x2,0}\tau^2 = 2a_{x,2}, \\ u_{x1,T}(T - \tau) + u_{x1,0}\tau = b_{x,1} - v_{x,1}, \\ u_{x2,T}(T - \tau) + u_{x2,0}\tau = b_{x,2} - v_{x,2}. \end{cases} \quad (14)$$

Based on (14) system, $u_{x,1,0}, u_{x,2,0}, u_{x,1,T}, u_{x,2,T}$ quantities can be expressed depending on τ and T :

$$\begin{cases} u_{x1,0} = \tau^{-1}T^{-1}[2a_{x,1} + (T + \tau)(b_{x,1} - v_{x,1})], \\ u_{x2,0} = \tau^{-1}T^{-1}[2a_{x,2} + (T + \tau)(b_{x,2} - v_{x,2})], \\ u_{x1,T} = -(T - \tau)^{-1}T^{-1}[2a_{x,1} + \tau(b_{x,1} - v_{x,1})], \\ u_{x2,T} = -(T - \tau)^{-1}T^{-1}[2a_{x,1} + \tau(b_{x,2} - v_{x,2})]. \end{cases} \quad (15)$$

If we consider the expressions (15) in (11), considering the variables, the following system of nonlinear algebraic equations of the 4th order is obtained depending on τ and T :

$$\begin{cases} [2a_{x,1} + (T + \tau)(b_{x,1} - v_{x,1})]^2 + \\ + [2a_{x,2} + (T + \tau)(b_{x,2} - v_{x,2})]^2 = \tau^2 T^2 u_0{}^2. \\ [2a_{x,1} + \tau(b_{x,1} - v_{x,1})]^2 + \\ + [2a_{x,2} + \tau(b_{x,2} - v_{x,2})]^2 = (T - \tau)^2 T^2 u_0{}^2 \end{cases} \quad (16)$$

As can be seen from the system (16), the value of τ and T variables depends only on the initial data of the problem - known $\upsilon_{x,1}, \upsilon_{x,2}, a_{x,1}, b_{x,1}, a_{x,2}, b_{x,2}$ quantities and $c_{x,1}, c_{x,2}, e_{x,1}, e_{x,2}$. Given these quantities, the system of equations (16) can be solved by approximate calculation methods, for example, simple iterations or Newton's method. ([4, 5]).

IV. CONCLUSION

To address the optimal control problem of the UAV aiming to reach its target in the shortest possible time, a numerical algorithm is introduced. This algorithm determines the moments to change control and computes the expected reaching time. Using these derived values, we formulated equations to ascertain the optimal control functions for guiding the UAV efficiently.

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