Optimization of source parameters in dynamic network structure objects

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Abstract—**The optimization of parameters for concentrated sources that influence the operation of a dynamic network structure is the focus of this study. The network structure comprises numerous one-dimensional objects, each governed by a system of ordinary differential equations with nonlocal boundary conditions to describe their states. Sources affect individual points within these subobjects, as well as their connection points. The optimization process aims to determine the optimal locations and power levels of these sources, based on predefined target functionality criteria. We derive the necessary optimality conditions for the concentrated sources' parameters and present the results of numerical experiments using a test problem as an illustrative example.**

Keywords—sources; placement of sources; non-local conditions; optimality conditions; functional gradient.

I. Introduction

The optimization problem involving the placement and power allocation of concentrated sources [1 – 4] that impact the performance of a complex system has been thoroughly investigated. This system comprises numerous one-dimensional components, each characterized by a set of ordinary differential equations with nonlocal boundary conditions. These components are interconnected in arbitrary sequences, relying solely on the states at their respective starting and ending points.

We refer to such complex objects as dynamic objects with a network structure, drawing an analogy to [5], and the corresponding mathematical models as network models. These objects are conveniently represented as directed graphs. It is important to note that these graphs are typically incomplete, with most of the elements in the connection matrix set to zero. Non-zero elements in this matrix indicate connections between the initial and final states of individual blocks corresponding to adjacent links in the graph. The problem under consideration encompasses various challenges, notably including the optimal control of transient processes in the unsteady flow of liquids or gases within complex pipeline networks [2, 3]. The mathematical models governing these processes consist of subsystems of hyperbolic partial differential equations, each describing fluid movement within a distinct section of the pipeline. At the junctions between these sections, continuity of flow and material balance conditions are upheld, resulting in nonlocal boundary conditions. The application of the method of characteristics in either time or spatial variables (analogous to the decomposition method) transforms the problem of controlling material flow within the transportation network into the problem addressed in this article.

II. Statement of the Problem

We consider a complex object consisting of m links (blocks), randomly connected by their ends, the structure of which is conveniently represented in the form of a directed graph.

The set of all vertices of the graph will be denoted by. I ., and the set of links. (k,s) . of length l^{ks} with a beginning at the vertex $k \in I$ and an end at the vertex $s \in I$ will be denoted by $J = \{(k,s): k,s \in I\}$, $|I| = N$, $|J| = m$, $|I|$ indicates the number of elements of the set I **.**

Let the sets of links $J_i^+ = \{(j,i): j \in I_i^+\}$, $J_i^- = \{(i,j): j \in I_i^-\}$ – respectively entering and leaving the i th vertex, I_i^+ and I_i^- – the sets of vertices adjacent to the i th vertex, which are respectively the ends and beginnings of links from the set J_i, $J_i = J_i^+ \bigcup J_i^-$, $I_i = I_i^+ \bigcup I_i^-$. Let us denote

$|J_i^+| = |I_i^+| = \overline{n}_i$, $|J_i^-| = |I_i^-| = \underline{n}_i$, $\overline{n}_i + \underline{n}_i = n_i$, $i \in I$ **.** It's clear that

$$\sum_{i\in I} \underline{n}_i = \sum_{i\in I} \overline{n}_i = m, \quad \sum_{i\in I} n_i = 2m.$$

In practical applications, as a rule, the relation $n_i << N$, $i \in I$, holds, i.e. the number of vertices adjacent to any vertex is much less than the total number of vertices.

Each link in the graph is associated with an independent subobject (block). Let the state of each of the links $(k,i)\in J$, $k\in I_i^+$, $i\in I$ be described by a system of $\aleph$ linear nonautonomous ordinary differential equations

$$\frac{du^{ki}(x)}{dx}=A^{ki}(x)u^{ki}(x)+ +B^{ki}w^{ki}\delta(x-\xi^{ki})+f^{ki}(x), \quad x\in(0,l^{ki}), \tag{1}$$

with M_i, $M_i\le n_i\cdot\aleph$, linearly independent boundary conditions specified in undivided form

$$\sum_{k\in I_i^-} g_j^{ik}u^{ik}(0)+\sum_{k\in I_i^+} q_j^{ki}u^{ki}(l^{ki})=\mathrm{v}_j^i, \quad j=\overline{1,M_i}, \quad i\in I. \tag{2}$$

Here functions $u^{ki}(x)=u^{ki}(x;\upsilon)\in\mathrm{R}^{\aleph}$ characterizes the state of the (k,i) th link with length l^{ki} at the point $x\in\left[0,l^{ki}\right]$; $\upsilon=(w,\mathrm{v},\xi)-$ vector of parameters to be optimized, whose parameters are $w\in\mathrm{R}^{\mu}, w=(w^{ki}=(w_1^{ki},...,w_{\mu_{ki}}^{ki})\in\Omega_{w^{ki}}\subset\mathrm{R}^{\mu_{ki}}:k\in I_i^+,i\in I)$, $w_j^{ki}-$ j – th component μ_{ki} - dimensional (k,i) - th external source acting on the (k,i) th subsystem at the point $\xi^{ki}\in\Omega_{\xi^{ki}}\equiv\left[0,l^{ki}\right]$, $\xi=(\xi^{ki}:k\in I_i^+,i\in I)$, $\xi\in\mathrm{R}^{\bar m}$, $\bar m\le m$; $\mathrm{v}\in\mathrm{R}^M$, $\mathrm{v}=\left(\mathrm{v}^i\in\Omega_{\mathrm{v}^i}\subset\mathrm{R}^{M_i},i\in I\right)$, $\mathrm{v}^i=\left(\mathrm{v}_1^i,...,\mathrm{v}_{M_i}^i\right)^{\mathrm{T}}$, v_j^i- j - i is the component of the external source acting on the i th vertex. Let's denote $\mu=\sum_{i\in I}\sum_{k\in I_k^+}\mu_{ki}$, $M=\sum_{i=1}^{N}M_i$.

The given in the problem are: $A^{ki}(x)\ne\mathrm{const}$, $f^{ki}(x)-$ respectively $\aleph-$ dimensional square matrix and vector continuous at $x\in[0,l^{ki}]$ functions; $B^{ki}-$ $(\aleph\times\mu_{ki})$- dimensional scalar matrices; row vectors $g_j^{ik_s}=\left(g_{j,1}^{ik_s},...,g_{j,\aleph}^{ik_s}\right)$, $k_s\in I_i^-$, $s=\overline{1,\underline{n}_i}$, $q_j^{k_si}=\left(q_{j1}^{k_si},...,q_{j\aleph}^{k_si}\right)$, $k_s\in I_i^+$, $s=\overline{1,\overline{n}_i}$, $j=\overline{1,M_i}$, $i\in I$. If $B^{ki}=0_{\aleph\times\mu_{ki}}$, then this means that there are no sources in the (k,i)-th section.

In practical problems, depending on the sign of the source parameters w^{ki}, $k\in I_i^+$, $i\in I$, the external source at a point ξ^{ki} is called "outflow" or "inflow".

The total number of subsystems in equation (2.1) is equal to the number of links m in the network, each of which connects to adjacent links (blocks) in an arbitrary order solely through non-separated (nonlocal) boundary conditions, as described in equation (2.2). The total number of differential equations in system (2.1) is equal to $m\aleph$, and the number of boundary conditions in equation (2.2) should also be equal to M, ensuring compatibility between equations: $M=m\aleph$.

We will assume that the boundary value problem (1), (2) has a unique solution. This, as is known [9], depends only on matrices $A^{ki}(x)$, $k\in I_i^+$, vectors $g_j^{ik_s}$, $k_s\in I_i^-$, $s=\overline{1,\underline{n}_i}$, $q_j^{k_si}$, $k_s\in I_i^+$, $s=\overline{1,\overline{n}_i}$, $j=\overline{1,M_i}$, $i\in I$, and does not depend on other data involved in the problem, in particular, on unknown vectors w,v,ξ.

Based on practical considerations, restrictions are imposed on the values of the parameters w^{ki}, ξ^{ki}, v^i, $k\in I_i^+$, $i\in I$, optimized in the problem:

$$w^{ki}\in\Omega_{W^{ki}},\ \mathrm{v}^i\in\Omega_{V^i},\ \xi^{ki}\in\Omega_{\xi^{ki}}\equiv[0;l^{ki}], \quad \Omega^{ki}=\Omega_{W^{ki}}\times\Omega_{V^i}\times\Omega_{\xi^{ki}}. \tag{3}$$

We will assume that the sets of admissible values $\Omega_{W^{ki}}$, Ω_{V^i} are convex and compact. It is required to find such values of the vector components $\upsilon=(w,\mathrm{v},\xi)$ for which the functional

$$\Im(w,\mathrm{v},\xi)=\sum_{i\in I}\sum_{k\in I_i^+}\int_0^{l^{ki}}\left\|u^{ki}(x)-U^{ki}(x)\right\|_{R^{\aleph}}^2dx+ +\square(w,\mathrm{v},\xi),$$

$$\square(w,\mathrm{v},\xi)=\varepsilon_1\sum_{i\in I}\sum_{k\in I_i^+}\left\|w^{ki}-\hat w^{ki}\right\|_{R^{\mu_{ki}}}^2+ +\varepsilon_2\sum_{i\in I}\left\|\mathrm{v}^i-\hat{\mathrm{v}}^i\right\|_{R^{M_i}}^2+\varepsilon_3\sum_{i\in I}\sum_{k\in I_i^+}\left\|\xi^{ik}-\hat\xi^{ik}\right\|_{R^{v_{ki}\times m}}^2. \tag{4}$$

gets the minimum value.

The optimized finite-dimensional vector (w,ξ,v), which determines the parameters and locations of external sources, in real problems has a small dimension, despite the large dimension of the system of differential equations (2.1) itself.

III. METHODS AND RESULTS OF THE STUDY

The convexity and differentiability of the functional (2.5) are studied, formulas for the gradient of the functional are obtained, and the necessary conditions for optimality with respect to the parameters to be optimized are formulated.

Theorem 1. Let all the conditions imposed on the functions and parameters involved in the problem (1) - (4) be satisfied. The functional $\Im(w,\mathrm{v},\xi)$ is convex in w,v, for a fixed admissible vector ξ.

It is easy to prove that the functional $\Im(w,\mathrm{v},\xi)$ is not convex on ξ if the condition $B^{ki}w^{ki}\ne0$ is satisfied for at least one section $(k,i)\in J$, i.e. there are links that are influenced by external sources.

The differentiability of the functional (4) and the formulas obtained in the work for the components of its gradient over the optimized triple $\upsilon=(w,\mathrm{v},\xi)$ are investigated. Necessary optimality conditions are formulated in variational form for problem (1) – (4).

In many practical applications, external sources do not participate at all links and vertices of the object. In particular, on some links or vertices their values may be set and not optimized. In these cases, the corresponding components of the gradients of the functional $\mathrm{grad}_w\Im(w,\mathrm{v},\xi)$, $\mathrm{grad}_{\mathrm{v}}\Im(w,\mathrm{v},\xi)$, $\mathrm{grad}_{\xi}\Im(w,\mathrm{v},\xi)$ are not calculated and are taken equal to zero.

To determine the optimal values of $\upsilon = (w, \mathrm{v}, \xi)$, using formulas for calculating the gradient components of the functional of the problem (1) – (4), one can use effective first-order optimization methods, for example, the gradient projection method [6].

IV. CONCLUSION

The optimization problem for a network structure is examined, which is characterized by a system of differential equations featuring large ordinary derivatives and nonlocal boundary conditions. It is demonstrated that the conjugate problem shares the same characteristics as the direct problem. Furthermore, the expressions for the gradient components of the functional, based on source parameters, incorporate values of both direct and adjoint variables. These variables are exclusively defined at their respective vertices and blocks.

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