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About approach to solution of the traveling salesman problem based on the annealing method with the fuzziness of the time perception

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Abstract—**This paper investigates the use of fuzzy numbers and the annealing method to find a solution to the traveling salesman problem, which involves finding the shortest time route for a given set of cities. Fuzzy numbers are used to model the inaccuracy and uncertainty of input data, and an annealing method is proposed to find solutions. A comparison of the results of the TSP problem using crisp and fuzzy numbers using the annealing method was carried out. The results of numerical experiments are given, which show that the use of fuzzy numbers, in particular triangular and parabolic, with the annealing method leads to a significant improvement in the results of the TSP problem compared to the use of crisp numbers.**

Keywords—traveling salesman problem, annealing method, time fuzziness.

I. Introduction

The way decisions are made in society in many cases depends on the emotional state of a person. Feelings are like a reference point that is determined by a goal that is influenced by various factors. Emotions can be the reason for behavior that is appropriate for a particular situation, even when it is not the most efficient, but allows you to avoid any consequences that may arise from exceeding a certain time limit.

Special attention should be paid to these factors in the processes of formation and improvement of many theoretical ideas in the field of modeling human behavior, one of which is the adaptation of physical and mathematical models to real life. This makes it possible to combine the power of computational methods with the peculiarities of human behavior. Such tasks are common in the context of the application of artificial intelligence methods and algorithms, the creation of decision-making support systems, the resolution of resource allocation issues taking into account the human factor, etc.

Time is an important resource in activities involving human participation. Estimation of time intervals is fundamental to understanding time frames, even though the exact boundaries of the interval may not be defined until the process reaches a certain stage. Thus, a period of time is usually defined by an indefinite interval that can be roughly predicted given the nature of the passage of time, if the given limits of the interval are taken as a given. To measure time intervals, they can be expressed in phrases such as "quick response", "normal timing" or "long wait". This means that when solving problems that require verbal terms to refer to time, it is important to take time variation into account. It is clear that emotions have a great influence on the understanding of time in processes that involve a person [1].

In order to find the most successful or effective solution to problems, it is necessary to take into account factors that affect human emotions and, therefore, the speed of time perception, resource allocation and calendar planning. The paper proposes to develop an approach to the formalization of accounting for the flow of time based on fuzzy numbers and to apply it to solving certain fuzzy optimization problems related to taking into account the fuzzy perception of time arising from the subjectivity and irregularity of the time count.

II. Traveling Salesman Problem with Time Criteria

According to the content of the traveling salesman problem (TSP, Traveling Salesman Problem) [2], it is necessary to create a route of movement within a given set of interconnected points (bridges) that form the transport network of a particular region. A feature of the problem is that the route must contain all the points specified in the task, and each of the points must be visited no more than once. It is clear that such trips take a lot of time, so it is logical that it is necessary to plan the route in such a way that the distance to be covered, or the time to overcome it, is minimal (finding the path with the least cost can be also considered as a criterion).

The traveling salesman problem is a combinatorial problem that can be solved using mathematical programming methods. To reduce the problem to a general form, we number the cities by numbers (*1, 2, 3, ..., n*), and describe the traveling salesman's route by a cyclic permutation of numbers $t = (j_1, j_2, ..., j_n, j_1)$, where all $j_1, ..., j_n$ are different numbers. The number j_1, repeated from the very beginning and at the end, shows that the permutation is cyclic.

The set of cities can be considered as the vertices of some graph with given distances (or travel time) between all pairs of vertices c_{ij} that form the matrix $C=(c_{ij})$, $i,j=\overline{1,n}$. We assume that the matrix is symmetric. The formal problem then is to find the shortest route (in time) t that goes through each city and ends at the starting point. In this formulation, the problem is called the closed traveling salesman problem, which is a well-known mathematical integer programming problem.

Let us formulate a mathematical model of the TSP problem. Let $I=\{1,...,n\}$ be the set of vertex indices of the problem graph. The objective function is the total time of the route, including all the vertices of the task graph. The parameters of the problem are the elements of the matrix $C=(c_{ij})$, $i,j \in I$.

Shift tasks are elements of the binary matrix of transitions between vertices $X = \{x_{ij}\}$, $i, j \in I$, which are equal to 1 if there is an edge (v_i, v_j) in the constructed route for the task, 0 otherwise. The shortest route in terms of time is optimal:

$$\sum_{i \in I} \sum_{j \in I, j \neq i} c_{ij} x_{ij} \to \min, \qquad (1)$$

with constraints $\sum_{j \in I, j \neq i} x_{ij} = 1, \ i \in I,$

$$\sum_{i \in I, j \neq i} x_{ij} = 1, \ j \in I, \qquad (2)$$

$v_i - v_j + n x_{ij} \le n - 1, \ 1 \le i \ne j \le n.$

The last inequality ensures the connectivity of the vertex traversal route; it cannot consist of two or more unconnected parts.

The formulated optimization problem can be solved using a greedy algorithm or any of the available optimization algorithms such as genetic algorithm, tabu search, ant colony optimization, branch-and-bound algorithm, etc. [3].

A. Annealing method

One of the methods of solving the traveling salesman problem using the combinatorial optimization technique is the annealing method. By analogy with the annealing process of various physical materials, in which by raising its temperature to a high level and then gradually lowering it, the algorithm accidentally disturbs the output path ("heating") for further gradual lowering of the "temperature" [4].

When modeling the annealing process, the analog of temperature is the level of randomness, with the help of which changes are made to the path, which in the future improves in its duration. When the "temperature" of the process is high, changes occur to avoid the danger of reaching a local minimum, followed by control at the optimal value as the "temperature" is successively reduced. "Temperature" decays in a series of steps on an exponential decay curve, with each step the temperature being lower than before.

B. Fuzzy passage of time

In everyday life, expressions such as "almost six", "quite tall", "not short enough" are often used to define a certain size in an approximate format. As a result, this method of evaluation requires the formalization of insufficiently clearly defined evaluations for their practical application in mathematical models. For this purpose, you can use concepts that allow you to present the subjective or intuitive meaning of fuzzy concepts in a constructive way. One of these concepts of uncertainty formalization is fuzzy numbers [5].

Fuzzy numbers are used to obtain results in problems related to decision-making and analysis. Fuzzy numbers defined in the number space are an extension of real numbers and have their own properties that can be attributed to number theory. To understand fuzzy numbers and their subspecies - triangular and parabolic numbers, consider the concept of a fuzzy set.

Let E be a set with a finite or infinite number of elements. Let A be the set contained in E. Then the set of ordered pairs $(x, \mu_{\tilde{A}}(x))$ defines a fuzzy subset $\tilde{A}$ for E, where x – is a member of E, and $\mu_{\tilde{A}}(x)$ – degree of belonging of x to A. The set of elements from A for which $\mu_{\tilde{A}}(x) > 0$, form the carrier of a vague set (reference set).

A fuzzy number is a generalization of an ordinary real number. It refers to a connected set of possible values, where each possible value has its own weight between 0 and 1. Thus, a fuzzy number is a special case of a convex normalized fuzzy set in the space of real numbers. Among the possible types of fuzzy numbers, triangular and parabolic numbers were considered.

A fuzzy number $\tilde{A} = (a, b, c)$ is called a triangular fuzzy number if its membership function looks like this:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x < a; \\ (x-a)/(b-a), a \le x \le b; \\ (c-x)/(c-b), b \le x \le c; \\ 0, x > c. \end{cases} \qquad (3)$$

Above the triangular numbers (Fig. 1), you can determine the main arithmetic operations for further use in calculations.

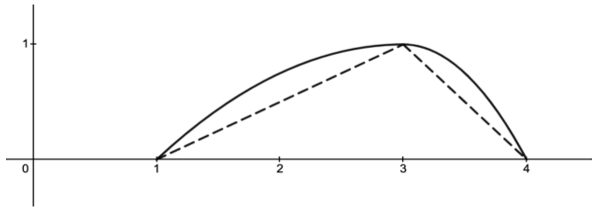

Figure 1. Example of triangle and parabolic fuzzy number

Let *A = (a, b, c)* and *B = (a1, b1, c1)* be two triangular numbers. Then

- the sum is defined as A + B = (a+a1, b+b1, c+c1);
- the difference is defined as *A - B = A + (-B) = =(a-c1, b-b1, c-a1)*, where *-B = (-c1, -b1, -a1)* is defined as the opposite of *B*.

In other words, opposite triangular numbers and their sum and difference are also triangular numbers. It is also worth noting that the results of inversion and multiplication of triangular numbers do not preserve this property and do not always represent triangular numbers.

The parabolic number (Fig. 1) is given similarly and has the same properties, but has a different membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x < a, \\ -((x-b)/(a-b))^2 + 1, a \le x \le b, \\ -((x-b)/(c-b))^2 + 1, b \le x \le c, \\ 0, x > c. \end{cases} \quad (4)$$

Although uncertainty information can be formalized using fuzzy numbers, the decision-making procedure must be clear. For example, the final output of fuzzy systems and the selection of appropriate solutions should be justified on the basis of the value characterized by the confidence (importance) indicator. To obtain a clear value, methods of calculating ranks (defuzzification) of fuzzy numbers are used, which are essentially clear representative numbers, and can be used as generalized values for further calculations. One of the methods for calculating the rank of a fuzzy number is the Jaeger method, which calculates the Jaeger rank of the first type in the form [6]

$$F1(\tilde{A}) = \int_0^1 g(x)\mu_{\tilde{A}}(x)dx \Big/ \int_0^1 \mu_{\tilde{A}}(x)dx,$$

where *g(x)* is a weight function that measures the importance of the value of *x*.

This method of defuzzification is also called the method of the center of gravity (COG) [7] (Fig. 2). Among other well-known methods, it is also worth noting the bisector of area (BOA) method [8], according to which there is such a value of *x* that a vertical line drawn through it divides the fuzzy number into two equal parts by their area.

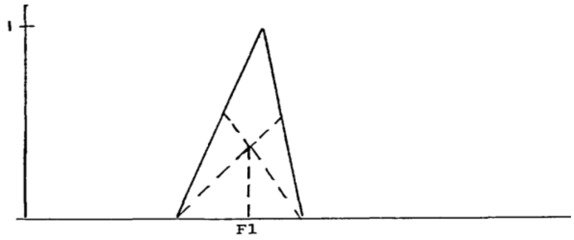

Figure 2. Defuzzification by method of the gravity center

III. Proposal

Given that the "cost" of travel between cities in time measurement can vary depending on the situation, a more accurate representation of such cost can be given in the form of triangular or parabolic numbers. If the subjective perception of time is chosen as the value, the relative duration of the trip between cities may vary depending on the factors affecting the path - traffic jams, bad weather, etc. Note that even in a simpler perception of the dynamic duration of the road between cities, when the actual time required to cover the path at the recommended average speed is measured, the same factors change the given duration, and therefore it makes sense to represent the studied travel time in the form of triangular or parabolic numbers.

Using one of the combinatorial methods of the approximate solution of the traveling salesman problem in combination with fuzzy numbers (and the corresponding method of calculating their rank), it is possible to achieve an effective result from the construction of the optimal path taking into account the dynamic features of roads between destinations. At the same time, better calculations can be obtained when using fuzzy parabolic numbers, since their essence is closer to reality. For the subjective overestimation or underestimation of the perception of the passage of time, the rule is valid: the greater the possible deviation in perception, the less likely it is to be obtained. For the numerical implementation of the actions of the annealing algorithm on fuzzy numbers that determine the time perception of the duration of movement between cities, operations according to the above schemes are used, and the different routes formed at the same time are compared with each other by finding and comparing the ranks of fuzzy numbers using one of the specified methods.

IV. Results

In the course of the study, the described algorithm was implemented based on the annealing method using fuzzy numbers to represent the subjective perception of the passage of time on road sections between cities. A multi-threaded Python implementation is proposed for numerical calculations. In the process of work, the method of calculating the rank of fuzzy numbers is chosen and the rank values of different methods (use of the peak abscissa, BOA, COG) are compared with the average value of random route passes. It was concluded that the best result was obtained by the center of gravity (COG) method. A comparison of the route estimation methods is given in the table, where the random route characterizes the time of travel along the route taking into account the average speed, the calculated route is the duration estimate obtained by the chosen method (see table I).

TABLE I. REZULTS

Method	Random route	Calculated route
The peak abscissa	5367.78	5046.0
BOA (triangular FN)	5332.54	5291.72
BOA (parabolic FN)	5369.91	5341.70
COG (triangular FN)	5332.40	**5332.86**
COG (parabolic. FN)	5369.67	**5368.72**

During the program processing, three possible approaches to finding solutions are compared (using crisp, triangular, and parabolic numbers, respectively). As initial conditions, the TSPLib library was used, which has known TSP conditions in its catalog in the form of arrays of coordinates or matrices of conditional distances between cities (e.g. u16, fr4, etc.). Fuzzy initial conditions were randomly generated with possible deviation from the expected value in either direction. To test the proposed approach, the time of the best constructed results for each type of fuzzy numbers was compared with the average value of the time taken for 10^5 random passes along the constructed route. The results of the work are shown in the table II:

TABLE II.

Task	Fuzzy number type	Estimated time	Actual time
u16	crisp	6859.0	7247.38
	triangular	6859.0	7129.38
	parabolic	6859.0	**7128.92**
gr4	crisp	5046.0	5369.57
	triangular	5071.0	5369.30
	parabolic	5070.0	**5363.88**
pr76	crisp	108273.0	115105.75
	triangular	108894.0	**114102.55**
	parabolic	109295.0	114563.41
rd100	crisp	8185.0	8653.50
	triangular	7975.0	**8390.85**
	parabolic	8049.0	8447.22
rd400	crisp	18070.0	19087.33
	triangular	18089.0	19008.33
	parabolic	17808.0	**18713.72**

Thus, it was concluded that the use of fuzzy numbers in the annealing algorithm allows to obtain constructive results when solving the traveling salesman problem with fuzzy input parameters.

V. CONCLUSIONS

This paper investigates the use of fuzzy numbers and the annealing method to find a solution to the traveling salesman problem, which involves finding the shortest time route for a given set of cities. Fuzzy numbers are used to model the inaccuracy and uncertainty of input data, and an annealing method is proposed to find solutions. The solutions obtained on the basis of the developed program in the Python language were analyzed. A comparison of the results of the TSP problem using crisp and fuzzy numbers using the annealing method was carried out. The results of numerical experiments are given, which show that the use of fuzzy numbers, in particular triangular and parabolic, with the annealing method leads to a significant improvement in the results of the TSP problem compared to the use of crisp numbers. This approach can be applied to real-world optimization problems involving imprecise or uncertain data and can be useful for optimizing processes with subjective time perception. A conclusion was made about the need for further research using the theory of fuzzy numbers, in particular in the direction of the correct choice of the type of numbers in accordance with the conditions of the task.

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