Thermal condition of cohesion in a two-layer roll of a rolling mill

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Abstract— A condition of heat exchange between the layers having different thermalphysic properties in a two-layer cylindrical roll of a rolling mill is analyzed for an ideal thermal contact. It can be realized with application of the condition of heat balance of one of the layers in the cylindrical area for a homogeneous equation of heat conductivity. Analyzed was a simplified target setting in the radial section with a supposition, regarding an averaged in radius temperature distribution in the outer layer. By applying the condition of the thermal balance and by integrating the homogeneous equation of heat conductivity in the two-layer area a condition of cohesion of an impedance type in case of an ideal thermal contact between the layers was constructed.

Keywords— mathematical simulatoin, heat conductivity equatuins, thermalconditoin of an impedance type.

I. INTRODUCTION

Contemporary requirements for the quality of production of rolled stock demand improved reliability of equipment operation, especially durability of rolls [8,9]. It can be realized by means of application of two-layer rolls, their outer layer being made of wear-resistant materials and their inner layer being made of heat-resistant materials. Durability of two-layer rolls can be increased by means of optimization of their thermal operation mode and control of the processes of heat exchange in roll’s pass with the help of a mathematical simulation model and the systems of computer mathematics [5,6]. The heat from the surface of a two-layer roll with a cylindrical shape is passed to its axis by means of heat conductivity [1-3]. Normally two-layer rolls of a rolling mill possess different thermophysics properties and there is a dense thermal contact between them. With a supposition of an existence of stationary heat exchange let us analyze the condition of thermal interaction at transition from one layer to another. Such thermophysics model describes the process of rolling product line in iron and steel industry and leads to a mathematical simulation model, based upon a homogeneous equation of heat conductivity, that has in the cylindrical coordinate system of the limited area the following view:

\[
\lambda \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} - c \rho \frac{\partial u}{\partial t} \right) = 0
\]

Now let us analyze the equation of heat balance in an arbitrary point of the area of \( \gamma \in \Omega \) (Fig. 1)

\[
\int_\gamma \left( \text{div} \left( \lambda \text{grad} u \right) - c \rho u \right) \, ds = 0.
\]

In our case area – is a section of the ring, located between the circles of \( R_1 \) and \( R_2 \) (\( R_2 > R_1 \)) radii and two radii that form angles \( \phi_1 \) and \( \phi_2 \) (\( \phi_2 > \phi_1 \)) with the start of the coordinates. Radius turn is performed from \( \phi_1 \) angle to \( \phi_2 \) angle (in a counterclockwise direction).

Let us denote

\[
u = \frac{u_+ + u_-}{2} + \frac{u_+ - u_-}{h} r = \bar{u} + \bar{u}' r
\]

\[
R_2 = R + \frac{h}{2}; \quad R_1 = R - \frac{h}{2}
\]

Having applied Ostrogradski’y’s formula let us now rearrange the first additive under the integral sign (1) [7],

\[
\int_\gamma \left( \lambda \text{grad} u \right) \, ds = \int_\gamma \lambda \frac{\partial u}{\partial r} \, dl,
\]

where \( \gamma \) - area contour.
Let us evaluate the integral along the boundary of $\gamma$ area.

$$
\int_\gamma \lambda \frac{\partial u}{\partial r} dl = \int_{\gamma_1} \lambda \frac{\partial u}{\partial r} dl + \int_{\gamma_2} \lambda \frac{\partial u}{\partial r} dl + \int_{\gamma_3} \lambda \frac{\partial u}{\partial r} dl + \int_{\gamma_4} \lambda \frac{\partial u}{\partial r} dl
$$

By passing to the polar coordinates $x = r \cos \phi$, $y = r \sin \phi$, at $\lambda = \text{const}$ we get:

$$
div(\lambda \text{grad} u) = \lambda \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \lambda \frac{\partial^2 u}{\partial \phi^2}
$$

or

$$
div(\lambda \text{grad} u) ds = \lambda \left[ \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial \phi^2} \right] r dr \phi =
$$

$$
= \lambda \int_{R_2^\phi} \frac{\partial u}{\partial (r \phi)} r dr \phi + \lambda \int_{R_2^\phi} \frac{\partial u}{\partial (r \phi)} r dr \phi +
$$

$$
+ \lambda \frac{\partial u}{\partial (r \phi)} r dr \phi + \lambda \frac{\partial u}{\partial (r \phi)} r dr \phi =
$$

$$
= \lambda \int_{R_2^\phi} \frac{\partial u}{\partial (r \phi)} r dr \phi + \lambda \frac{\partial u}{\partial (r \phi)} r dr \phi +
$$

$$
+ \lambda \frac{\partial u}{\partial (r \phi)} r dr \phi + \lambda \frac{\partial u}{\partial (r \phi)} r dr \phi =
$$

Thus, instead of (1) we get the equation

$$
\lambda \int_{R_2^\phi} \frac{\partial u}{\partial r} dl + \lambda \int_{R_3^\phi} \frac{\partial^2 u}{\partial \phi^2} dr
$$

Now, let us consider the third addition in the previous expression, using (2)

$$
\int_{R_2^\phi} \frac{\partial^2 u}{\partial \phi^2} (r, \phi) dr d\phi = \int_{R_2^\phi} \frac{\partial^2 u}{\partial \phi^2} (r, \phi) dr d\phi
$$

By applying the theorem regarding the average, we can get

$$
\lambda (\varphi_2 - \varphi_1) R_2 \frac{\partial u}{\partial r} \bigg|_{R_2} - \lambda (\varphi_2 - \varphi_1) R_1 \frac{\partial u}{\partial r} \bigg|_{R_1} +
$$

$$
\int_{R_2^\phi} \frac{\partial^2 u}{\partial \phi^2} (r, \phi) dr d\phi
$$

where $1 \frac{\partial^2}{\partial \phi^2} = \Delta_{\phi}$. 
+\lambda r^2 (\varphi_2 - \varphi_1) \Delta \omega \left( \pi \ln \frac{R + \frac{h}{2}}{R - \frac{h}{2}} + \pi h \right) - c \rho (\varphi_2 - \varphi_1) \left[ \bar{u} R h + \bar{u} \left( R^2 h + \frac{h^3}{12} \right) \right] = 0.

Both sections of the last equation we will divide by \( \varphi_2 - \varphi_1 \):

\[
R_2 \lambda \frac{\partial u}{\partial r} \left| _{R_2} - R_1 \lambda \frac{\partial u}{\partial r} \left| _{R_1} + \lambda \frac{\partial^2 u}{\partial \varphi^2} \left( \pi \ln \frac{R + \frac{h}{2}}{R - \frac{h}{2}} + \pi h \right) - c \rho \left[ \bar{u} R h + \bar{u} \left( R^2 h + \frac{h^3}{12} \right) \right] = 0. \tag{4}
\]

\[
\left( R + \frac{h}{2} \right) \lambda \frac{\partial u}{\partial r} \left| _{R + \frac{h}{2}} - \left( R - \frac{h}{2} \right) \lambda \frac{\partial u}{\partial r} \left| _{R - \frac{h}{2}} + \lambda \frac{\partial^2 u}{\partial \varphi^2} \left( \pi \ln \frac{R + \frac{h}{2}}{R - \frac{h}{2}} + \pi h \right) - c \rho \left[ \bar{u} R h + \bar{u} \left( R^2 h + \frac{h^3}{12} \right) \right] = 0. \tag{5}
\]

The formulae (4) and (5) are condition of cohesion of an impedance type for a cylinder.

Now, let us consider a simplified target setting. Let one part of the surface of a two-layer cylinder, rotating around its axis with constant velocity \( \omega \) is heated by a constant heat flow the density of which is \( \bar{w} \), \( \omega < \varphi < \omega + \Omega \) while the other part radiates heat into the ambient space in accordance with the laws of Newton and S. Bolzman. For determination of temperature distribution \( T = T(r, \varphi, z, t) \) in such cylinder on the assumption that temperature distribution of the outer layer does not depend upon its radius we can pass to the analysis of the averaged temperature field along the radius in the outer layer. Thus, we can reach the following boundary problem regarding adhesion in the area:

\[
\Omega \times t = \left\{ (r, z, \varphi, t) \mid 0 < r < R_2, 0 < \varphi < 2 \pi, 0 < z < l, t > 0 \right\}
\]

\[
\lambda_2 \frac{C}{r} \frac{\partial T}{\partial r} + \lambda_2 \frac{C}{r^2} \frac{\partial^2 T}{\partial \varphi^2} + \lambda_2 \frac{\partial^2 T}{\partial z^2} - c_{1,2} \rho_1 \lambda_2 \frac{\partial T}{\partial t} = 0 \tag{6}
\]

\[
T(r, \varphi, z, 0) = T_0, \tag{7}
\]

\[
T(r, \varphi, 0, t) = T_0, \quad T(r, \varphi, l, t) = T_0, \quad T(r, \varphi, z, t) = \lambda_1 T_0 (R_1 - 0, \varphi, z, t), \tag{8}
\]

\[
\lambda_2 T(R_2 + 0, \varphi, z, t) = \lambda_2 T_0 R_2 - 0, \varphi, z, t), \tag{9}
\]

\[
\lambda_2 \frac{\partial T(R_2, \varphi, z, t)}{\partial r} = \alpha \left( T_z - T \right) + \rho_2 (T_C^4 - T^4), \tag{10}
\]

\[
\omega t + \varphi_0 < \omega < \omega + 2 \pi \tag{11}
\]

\[
\lambda_2 \frac{\partial T(R_2, \varphi, z, t)}{\partial r} = -\bar{u}, \quad \omega t + \varphi < \omega + \varphi_0, \tag{12}
\]

\[
\frac{\partial T(0, \varphi, z, t)}{\partial r} = 0, \tag{13}
\]

\[
T(r, \varphi + 2 \pi, z, t) = T(r, \varphi, z, t), \tag{14}
\]

where \( \bar{w} \) – being the heat flow from the stripe to the rolls in the area of the contact of the stripe with rolls \( R_1 < R_2 \). At such conditions of heat exchange we can assume in the mathematical simulation model that the temperature of the cylinder along its length remains constant, so the derivative along \( z \) can be neglected, assuming that \( \frac{\partial T}{\partial z} = 0. \) At that the problem (6)-(14) is simplified, the condition (8) disappears. Let one consider the simplified conditions (8) of heat exchange we can assume in the mathematical simulation model that the temperature of the cylinder along its length remains constant, so the derivative along \( z \) can be neglected, assuming that \( \frac{\partial T}{\partial z} = 0. \) At that the problem (6)-(14) is simplified, the condition (8) disappears. For investigation of the temperature field of the inner cylinder it's enough to know the average temperature distribution in the outer one (Fig. 1). So, for evaluation of the heat flow through the surface of the inner cylinder we can multiply the equation and (15) by \( rdr \) and integrate along the layer's thickness within from \( R_1 \) to \( R_2 \).

By applying the relation

\[
u(\varphi, t) = \frac{2}{S} \int_{R_1}^{R_2} T(r, \varphi, t) rdr, \tag{16}
\]

where \( S \) – is the area of the perimeter of the outer layer, we will get after rearranging
\[ R_2 \int \left( \lambda_2 \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \lambda_2 \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} - c_2 \rho_2 \frac{\partial T}{\partial t} \right) r dr = 0 \]

\[ R_2 \int \left( \lambda_2 \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) r dr + \lambda_2 \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} r dr - c_2 \rho_2 \frac{\partial T}{\partial t} r dr = 0 \]

\[ \lambda_2 \left( \frac{\partial T}{\partial r} \right) \bigg|_{r=R_2} + \lambda_2 \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} \int_{R_1}^{R_2} r dr - c_2 \rho_2 \frac{\partial T}{\partial t} \int_{R_1}^{R_2} r dr = 0 \]

By dividing both parts of the equation by \( \lambda_2 R_1 \), we will receive that

\[ \frac{\partial T}{\partial r} - 2 \frac{\rho_2}{\lambda_2 R_1} \frac{\partial T}{\partial t} = \]

\[ = \frac{R_2}{R_1} \left( \alpha(T_c - T) + \varepsilon \sigma(T_c^4 - T^4) \right) \]

\[ g_1(\alpha T) + g_2(\sigma T) = \frac{R_2}{R_1} \left( \alpha(T_c - T) + \varepsilon \sigma(T_c^4 - T^4) \right) \]

Let \( \alpha_1 = \frac{S}{2 \rho_2 R_1} \), \( \alpha_2 = \frac{S \rho_2}{\lambda_2 R_1} \), \( g_0 = \frac{R_2}{R_1} \frac{\partial T}{\partial r} \).

\[ g_0 \cdot \left( \alpha T \right) + g_2 \left( \sigma T \right) = \frac{R_2}{R_1} \left( \alpha(T_c - T) + \varepsilon \sigma(T_c^4 - T^4) \right) \]

where \( g_1 = \text{const}, g_0, g_1 (\alpha T), g_2 (\sigma T) \) - are constant values or partial-monotonous functions.

The obtained boundary condition of cohesion of an impedance type (18) is but a particular case of a more complicated condition (5). Both conditions (18) and (5) seem to be suitable for applying for solving boundary problems for multi-layer cylindrical areas, particularly for investigation of temperature distributions in multi-layer rolls of rolling mills and rolled crystallizers.

II. CONCLUSIONS

Obtained in the work was the condition of thermal cohesion between the layers of a rolling mill rolls of cylindrical shape, possessing different thermal physics properties of their layers at dense thermal contact of the layers. This condition was set up with application of the condition of the heat balance of the area that includes the boundary of separation of the layers. The objective of obtaining this condition was in the necessity of developing and investigating a mathematical model of heat distribution in the rolls when a metal stripe or sections are being rolled. The reason was a necessity of designing new advanced process equipment. On the basis of the equation of heat balance of an element of two-layer cylindrical area and the boundary condition of the fourth order a condition of an impedance type was developed, which has a tangential derivative along with the normal derivative. Such condition allows to investigate temperature distributions in multi-layer and complicated noncharacteristic areas.

REFERENCES