Thermal condition of cohesion in a two-layer roll of a rolling mill

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Abstract— **A condition of heat exchange between the layers having different thermalphysic properties in a two-layer cylindrical roll of a rolling mill is analyzed foe an ideal thermal contact. It can be realized with application of the condition of heat balance of one of the layers in the cylindrical area for a homogeneous equation of heat conductivity. Analyzed was a simplified target setting in the radial section with a supposition, regarding an averaged in radius temperature distribution in the outer layer. By applying the condition of the thermal balance and by integrating the homogeneous equation of heat conductivity in the two-layer area a condition of cohesion of an impedance type in case of an ideal thermal contact between the layers was constructed.**

Keywords— mathematical simulatoin, heat conductivity equatuins, thermalconditoin of an impedance type.

I. INTRODUCTION

Contemorary requirements for the quality of production of rolled stock demand improved reliability of equipmnet operatoin, especially durability of rolls [8,9]. It can be realized by means of application of two-layer rolls, their outer layer being made of wear-resistant materials and their innerv layer being made of heat-resistant materials. Durability of two-layer rolls can be increased by means of optimization of their thermal operatoin mode and control of the processes of heat exchange in roll's pass with the help of a mathematical simulatoin model and the systems of computer mathematics [5,6]. The heat from the surface of a two-layer roll with a cylindrical shape is passed to its axis by means of heat conductivity[1-3]. Normally two-layer rolls of a rolling mill possess different thermalphysics properties and there is a dense thermal contact between them. With a suppositoin of an existence of stationary heat exchange let us analyze the condition of thermal interaction at transitoin from one layer to another. Such thermalphysics model describes the process of rolling productoin in iron and steel industry and leads to a mathematical simulation model, based upon a homogeneous equation of heat conductivity, that has in the cylindrical coordinate system of the limited Ω area the following view:

$$\lambda_i \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\lambda_i \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}+\lambda_i \frac{\partial^2 u}{\partial z^2} - c_i\rho_i \frac{\partial u}{\partial t}=0$$

Now let us analyze the equation of heat balance in an arbitrary point of the area of $\gamma \in \Omega$ (Fig. 1)

$$\int_{\gamma}\left[div\left(\lambda\, grad\, u\right)-c\rho u_t\right]ds = 0. \qquad (1)$$

In our case γ area – is a section of the ring, located between the circles of R_2 i R_1 $\left(R_2 > R_1\right)$ radii and two radii that form angles φ_1 i φ_2 $\left(\varphi_2 > \varphi_1\right)$ with the start of the coordinates. Radius turn is performed from φ_1 angle to φ_2 angle (in a counterclockwise direction).

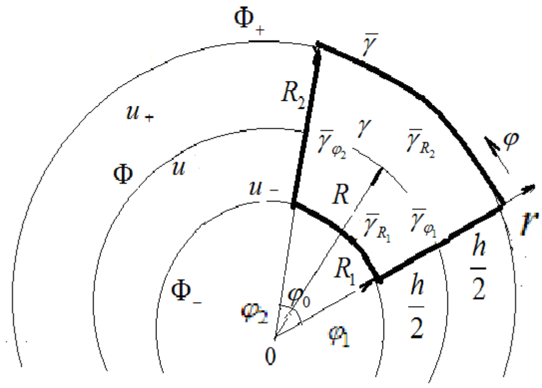

Figure 1. Area γ

Let us denote

$$u=\frac{u_+ + u_-}{2}+\frac{u_+ - u_-}{h}r=\bar{u}+\bar{u}'r \qquad (2)$$

$$R_2 = R+\frac{h}{2}; \qquad R_1 = R-\frac{h}{2} \qquad (3)$$

Having applied Ostrogradskiy's formula let us now rearrange the first additive under the integral sign (1) [7],

$$\int_{\gamma} div\left(\lambda grad\, u\right)ds = \int_{\bar{\gamma}} \lambda\frac{\partial u}{\partial r}dl,$$

where $\bar{\gamma}$ - γ area contour.

Let us evaluate the integral along the boundary of γ area.

$$\int_{\bar\gamma} \lambda \frac{\partial u}{\partial r} dl = \int_{\bar\gamma_{R_1}} \lambda \frac{\partial u}{\partial r} dl + \int_{\bar\gamma_{R_2}} \lambda \frac{\partial u}{\partial r} dl + \int_{\bar\gamma_{\varphi_1}} \lambda \frac{\partial u}{\partial r} dl + \int_{\bar\gamma_{\varphi_2}} \lambda \frac{\partial u}{\partial r} dl$$

By passing to the polar coordinates $x = r\cos\varphi$, $y = r\sin\varphi$, at $\lambda = const$ we will get :

$$div(\lambda grad\, u) = \lambda \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \lambda \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}$$

or $\int_\gamma div(\lambda grad\, u) ds = \lambda \int_\gamma \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \varphi^2}\right] r dr d\varphi =$

$$= \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u}{\partial(-r)} r\Big|_{r=R_1} d\varphi + \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u}{\partial r} r\Big|_{r=R_2} d\varphi +$$

$$+ \lambda \int_{R_1}^{R_2} \frac{\partial u}{\partial(-\varphi)} \frac{1}{r}\Big|_{\varphi=\varphi_1} dr + \lambda \int_{R_1}^{R_2} \frac{\partial u}{\partial \varphi} \frac{1}{r}\Big|_{\varphi=\varphi_2} dr =$$

$$= -\lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u_-}{\partial r} R_1 d\varphi + \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u_+}{\partial r} R_2 d\varphi -$$

$$- \lambda \int_{R_1}^{R_2} \frac{1}{r}\frac{\partial u}{\partial \varphi}\Big|_{\varphi=\varphi_1} dr + \lambda \int_{R_1}^{R_2} \frac{1}{r}\frac{\partial u}{\partial \varphi}\Big|_{\varphi=\varphi_2} dr =$$

$$= \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u_+}{\partial r} R_2 d\varphi - \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u_-}{\partial r} R_1 d\varphi +$$

$$+ \int_{R_1}^{R_2} \frac{1}{r}\left(\frac{\partial u}{\partial \varphi}\Big|_{\varphi=\varphi_2} - \frac{\partial u}{\partial \varphi}\Big|_{\varphi=\varphi_1}\right) dr$$

$$= \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u_+}{\partial r} R_2 d\varphi - \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u_-}{\partial r} R_1 d\varphi + \lambda \int_{R_1}^{R_2} \frac{1}{r}\frac{\partial u}{\partial \varphi}\Big|_{\varphi_1}^{\varphi_2} dr =$$

$$= \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u(R_2,\varphi)}{\partial r} R_2 d\varphi - .$$

$$-\lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u(R_1,\varphi)}{\partial r} R_1 d\varphi + \lambda \int_{\varphi_1}^{\varphi_2}\int_{R_1}^{R} \frac{1}{r}\frac{\partial^2 u(r,\varphi)}{\partial \varphi^2} dr d\varphi$$

Now, let us consider the third addition in the previous expression, using (2)

$$\int_{\varphi_1}^{\varphi_2}\int_{R_1}^{R_2} \frac{1}{r}\frac{\partial^2 u(r,\varphi)}{\partial \varphi^2} dr d\varphi = \int_{\varphi_1}^{\varphi_2} \frac{\partial^2}{\partial \varphi^2}\left(\int_{R_1}^{R_2} \frac{1}{r} u(r,\varphi) dr\right) d\varphi =$$

$$= \int_{\varphi_1}^{\varphi_2} \frac{\partial^2}{\partial \varphi^2} \int_{R_1}^{R_2} \frac{1}{r}(\bar u + \bar u' r) dr d\varphi = \int_{\varphi_1}^{\varphi_2} \frac{\partial^2}{\partial \varphi^2} \int_{R_1}^{R_2} \left(\frac{1}{r}\bar u + \bar u'\right) dr d\varphi =$$

$$= \int_{\varphi_1}^{\varphi_2} \frac{\partial^2}{\partial \varphi^2}(\bar u \ln r + \bar u' r)\Big|_{R_1}^{R_2} d\varphi = \int_{\varphi_1}^{\varphi_2} \frac{\partial^2}{\partial \varphi^2}(\bar u \ln r + \bar u' r)\Big|_{R-\frac{h}{2}}^{R+\frac{h}{2}} d\varphi =$$

$$= \int_{\varphi_1}^{\varphi_2} \frac{\partial^2}{\partial \varphi^2}\left(\bar u \ln \frac{R+\frac{h}{2}}{R-\frac{h}{2}} + \bar u' h\right) d\varphi =$$

$$= r_{cp}^2 \int_{\varphi_1}^{\varphi_2} \Delta_\varphi \left(u_+\left(\frac{1}{2}\ln\frac{R+\frac{h}{2}}{R-\frac{h}{2}} + 1\right) + u_-\left(\frac{1}{2}\ln\frac{R+\frac{h}{2}}{R-\frac{h}{2}} - 1\right)\right) d\varphi ,$$

where $\frac{1}{r_s^2}\frac{\partial^2}{\partial \varphi^2} = \Delta_\varphi$.

Now, let us evaluate the second additive with (1) with regard to (2) and (3)

$$\int_\gamma c\rho u_t ds = \int_{R_1}^{R_2}\int_{\varphi_1}^{\varphi_2} c\rho u_t r\, dr\, d\varphi = c\rho \int_{\varphi_1}^{\varphi_2}\int_{R_1}^{R_2} (\dot{\bar u} + \dot{\bar u}' r) r\, dr\, d\varphi =$$

$$c\rho \int_{\varphi_1}^{\varphi_2}\int_{R_1}^{R_2} (\dot{\bar u} r + \dot{\bar u}' r^2) dr d\varphi = c\rho \int_{\varphi_1}^{\varphi_2} \left(\dot{\bar u}\frac{r^2}{2} + \dot{\bar u}'\frac{r^3}{3}\right)\Big|_{R-\frac{h}{2}}^{R+\frac{h}{2}} d\varphi =$$

$$= c\rho \int_{\varphi_1}^{\varphi_2} \left\{\frac{\dot{\bar u}}{2}\left[\left(R+\frac{h}{2}\right)^2 - \left(R-\frac{h}{2}\right)^2\right] + \right.$$

$$\left. + \frac{\dot{\bar u}'}{3}\left[\left(R+\frac{h}{2}\right)^3 - \left(R-\frac{h}{2}\right)^3\right]\right\} d\varphi =$$

$$= c\rho \int_{\varphi_1}^{\varphi_2} \left(\frac{\dot{\bar u}}{2}\cdot 2Rh + \frac{\dot{\bar u}}{3}\left(3R^2 h + \frac{h^3}{4}\right)\right) d\varphi =$$

$$= c\rho \int_{\varphi_1}^{\varphi_2} \left[\dot{\bar u} Rh + \dot{\bar u}'\left(R^2 h + \frac{h^3}{12}\right)\right] d\varphi.$$

Thus, instead of (1) we get the equation

$$\lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u}{\partial r}\Big|_{r=R_2} R_2 d\varphi - \lambda \int_{\varphi_1}^{\varphi_2} \frac{\partial u}{\partial r}\Big|_{r=R_1} R d\varphi +$$

$$+ \lambda r_s^2 \int_{\varphi_1}^{\varphi_2} \Delta_\varphi \left[u_+\left(\frac{1}{2}\ln\frac{R+\frac{h}{2}}{R-\frac{h}{2}} + 1\right) + u_-\left(\frac{1}{2}\ln\frac{R+\frac{h}{2}}{R-\frac{h}{2}} - 1\right)\right] d\varphi -$$

$$c\rho \int_{\varphi_1}^{\varphi_2} \left[\dot{\bar u} Rh + \dot{\bar u}'\left(R^2 h + \frac{h^3}{12}\right)\right] d\varphi = 0.$$

By applying the theorem regarding the average, we can get

$$\lambda(\varphi_2 - \varphi_1) R_2 \frac{\partial u}{\partial r}\Big|_{R_2} - \lambda(\varphi_2 - \varphi_1) R_1 \frac{\partial u}{\partial r}\Big|_{R_1} +$$

$$+\lambda r_s^2(\varphi_2-\varphi_1)\Delta_\varphi\left(\bar{u}\ln\frac{R+\frac{h}{2}}{R-\frac{h}{2}}+\bar{u}'h\right)-$$

$$-c\rho(\varphi_2-\varphi_1)\left[\dot{\bar{u}}Rh+\dot{\bar{u}}'\left(R^2h+\frac{h^3}{12}\right)\right]=0.$$

Both sections of the last equation we will divide by $(\varphi_2-\varphi_1)$:

$$R_2\lambda\left.\frac{\partial u}{\partial r}\right|_{R_2}-R_1\lambda\left.\frac{\partial u}{\partial r}\right|_{R_1}+\lambda\frac{\partial^2 u}{\partial\varphi^2}\left(\bar{u}\ln\frac{R+\frac{h}{2}}{R-\frac{h}{2}}+\bar{u}'h\right)-$$

$$-c\rho\left[\dot{\bar{u}}Rh+\dot{\bar{u}}'\left(R^2h+\frac{h^3}{12}\right)\right]=0\,. \quad (4)$$

$$\left(R+\frac{h}{2}\right)\lambda\left.\frac{\partial u}{\partial r}\right|_{R+\frac{h}{2}}-\left(R-\frac{h}{2}\right)\lambda\left.\frac{\partial u}{\partial r}\right|_{R-\frac{h}{2}}+$$

$$+\lambda\frac{\partial^2 u}{\partial\varphi^2}\left(\bar{u}\ln\frac{R+\frac{h}{2}}{R-\frac{h}{2}}+\bar{u}'h\right)- \quad (5)$$

$$c\rho\left[\dot{\bar{u}}Rh+\dot{\bar{u}}'\left(R^2h+\frac{h^3}{12}\right)\right]=0\,.$$

The formulae (4) and (5) – are condition of cohesion of an impedance type for a cylinder.

Now, let us consider a simplified target setting. Let one part of the surface of a two-layer cylinder, rotating around its axis with constant velocity ω is heated by a constant heat flow the density of which is $\bar{w}$, $\omega t<\varphi<\varphi_0+\omega t$ while the other part radiates heat into the ambient space in accordance with the laws of Newton and S. Bolzman. For determination of temperature distribution $T=T(r,\varphi,z,t)$ in such cylinder on the assumption that temperature distribution of the outer layer does not depend upon its radius we can pass to the analysis of the averaged temperature field along the radius in the outer layer. Thus, we can reach the following boundary problem regarding adhesion in the area:

$$\Omega\times t=\{(r,z,\varphi,t)|\ 0<r<R_2, 0<\varphi<2\pi, 0<z<l, t>0\}$$

$$\lambda_{1,2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\lambda_{1,2}\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}+\lambda_{1,2}\frac{\partial^2 T}{\partial z^2}-c_{1,2}\rho_{1,2}\frac{\partial T}{\partial t}=0 \quad (6)$$

$$T(r,\varphi,z,0)=T_0, \quad (7)$$

$$T(r,\varphi,0,t)=T_0, \qquad T(r,\varphi,l,t)=T_0, \quad (8)$$

$$\lambda_1 T_r(R_1+0,\varphi,z,t)=\lambda_2 T_r(R_2-0,\varphi,z,t), \quad (9)$$

$$\lambda_1 T(R_1+0,\varphi,z,t)=\lambda_2 T(R_2-0,\varphi,z,t), \quad (10)$$

$$\lambda_2\frac{\partial T(R_2,\varphi,z,t)}{\partial r}=\alpha(T_c-T)+\varepsilon\sigma(T_c^4-T^4), \quad (11)$$
$$\omega t+\varphi_0<\varphi<\omega t+2\pi$$

$$\lambda_2\frac{\partial T(R_2,\varphi,z,t)}{\partial r}=-\bar{w},\quad \omega t<\varphi<\omega t+\varphi_0, \quad (12)$$

$$\frac{\partial T(0,\varphi,z,t)}{\partial r}=0, \quad (13)$$

$$T(r,\varphi+2\pi,z,t)=T(r,\varphi,z,t), \quad (14)$$

where $\bar{w}$ – being the heat flow from the stripe to the rolls in the area of the contact of the stripe with rolls $R_1<R_2$. At such conditions of heat exchange we can assume in the mathematical simulation model that the temperature of the cylinder along its length remains constant , so the derivative along z can be neglected, assuming that $\frac{\partial T}{\partial z}=0$. At that the problem (6)-(14) is simplified, the condition (8) disappears, while the equation (6) acquires in the area of $\Omega_1\times t=\{(r,\varphi,t)|\ 0<r<R_2, 0<\varphi<2\pi, t>0\}$ the following view:

$$\lambda_{1,2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\lambda_{1,2}\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}-c_{1,2}\rho_{1,2}\frac{\partial T}{\partial t}=0 \quad (15)$$

For investigation of the temperature field of the inner cylinder it's enough to know the average temperature distribution in the outer one (See Fig. 1). So, for evaluation of the heat flow through the surface of the inner cylinder we cam multiply the equation and (15) by rdr and integrate along the layer's thickness within from R_1 to R_2.

By applying the relation

$$u(\varphi,t)=\frac{2}{S}\int_{R_1}^{R_2}T(r,\varphi,t)rdr\ , \quad (16)$$

where S – is the area of the perimeter of the outer layer , we will get after rearranging

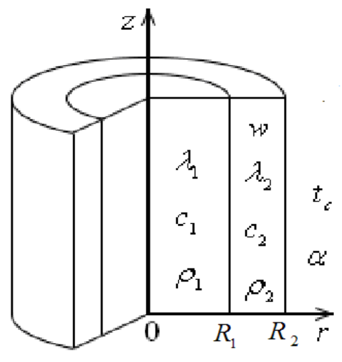

Figure. 2 Two-layer cylinder

$$\int_{R_1}^{R_2}\left(\lambda_2\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})+\lambda_2\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}-c_2\rho_2\frac{\partial T}{\partial t}\right)rdr=0$$

$$\lambda_2\int_{R_1}^{R_2}\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})rdr+\lambda_2\int_{R_1}^{R_2}\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}rdr-c_2\rho_2\int_{R_1}^{R_2}\frac{\partial T}{\partial t}rdr=0$$

$$\lambda_2(r\frac{\partial T}{\partial r})\Big|_{R_1}^{R_2}+\lambda_2\int_{R_1}^{R_2}\frac{1}{r^2}\frac{\partial^2 T}{\partial\varphi^2}rdr-c_2\rho_2\int_{R_1}^{R_2}\frac{\partial T}{\partial t}rdr=0. \quad (17)$$

Paying attention to the generalized theorem regarding the average value and taking into account (11)–(13), we may write down

$$\left(\lambda_2 R_2\left(\left(\alpha(T_c-T)+\varepsilon\sigma\left(T_c^4-T^4\right)\right)\Big|_{\omega t+\varphi_0<\varphi<\omega t+2\pi}\right)-\right.$$

$$\left.-w\big|_{\omega t<\varphi<\omega t+\varphi_0}\right)-$$

$$\left.-\lambda_2 R_1\frac{\partial T}{\partial r}\right)+\frac{\lambda_2}{r_s^2}\frac{\partial^2}{\partial\varphi^2}\int_{R_1}^{R_2}Trdr-c_2\rho_2\frac{\partial}{\partial t}\int_{R_1}^{R_2}Trdr=0$$

By dividing both parts of the equation by $\lambda_2 R_1$, we will receive that

$$\frac{\partial T}{\partial r}-\frac{S}{2r_s^2R_1}\frac{\partial^2 T}{\partial\varphi^2}+\frac{Sc_2\rho_2}{2\lambda_2R_1}\frac{\partial T}{\partial t}=$$

$$=\frac{R_2}{R_1}\begin{cases}-\bar{w}, & \omega t<\varphi<\omega t+\varphi_0,\\ \alpha(T_c-T)+\varepsilon\sigma\left(T_c^4-T^4\right), & \omega t+\varphi_0<\varphi<\omega t+2\pi\end{cases}$$

Let $\alpha_1=\frac{S}{2r_s^2R_1}$, $\alpha_2=\frac{Sc_2\rho_2}{\lambda_2 2R_1}$, $g_0=-\frac{R_2}{R_1}\bar{w}$,

$g_1(\alpha T)+g_2(\sigma T)=\frac{R_2}{R_1}\left(\alpha(T_c-T)+\varepsilon\sigma\left(T_c^4-T^4\right)\right)$, then, after our transformations we will have for the boundary of the inner and outer cylinders the condition of cohesion of an impedance type

$$\frac{\partial T}{\partial r}-\alpha_1\frac{\partial^2 T}{\partial\varphi^2}+\alpha_2\frac{\partial T}{\partial t}\Big|_{r=R_2}=$$

$$=\begin{cases}g_0, & \omega t<\varphi<\omega t+\varphi_0,\\ g_1(\alpha T)+g_2(\sigma T), & \omega t+\varphi_0<\varphi<\omega t+2\pi.\end{cases} \quad (18)$$

where $\alpha_i=const$, g_0, $g_1(\alpha T)$, $g_2(\sigma T)$ – are constant values or partial-monotonous functions.

The obtained boundary condition of cohesion of an impedance type (18) is but a particular case of a more complicated condition (5). Both conditions (18) and (5) seem to be suitable for applying for solving boundary problems for multi-layer cylindrical areas, particularly for investigation of temperature distributions in multi-layer rolls of rolling mills and rolled crystallizers.

II. CONCLUSIONS

Obtained in the work was the condition of thermal cohesion between the layers of a rolling mill rolls of cylindrical shape, possessing different thermalphysics properties of their layers at dense thermal contact of the layers. This condition was set up with application of the condition of the heat balance of the area that includes the boundary of separation of the layers. The objective of obtaining this condition was in the necessity of developing and investigating a mathematical model of heat distribution in the rolls when a metal stripe or sections are being rolled. The reason was a necessity of designing new advanced process equipment. On the basis of the equation of heat balance of an element of two-layer cylindrical area and the boundary condition of the fourth order a condition of an impedance type was developed, which has a tangential derivative along with the normal derivative. Such condition allows to investigate temperature distributions in multi-layer and complicated noncharacteristic areas.

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