Modeling of nonlinear clarification of aqueous suspensions at rapid filters

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Abstract—**The basic mathematical problem of nonlinear deep bed clarification of aqueous suspensions at rapid filters is formulated. The results of its solution using exact method for the linear form of the adhesion function and approximate method for the nonlinear form are selectively presented. The derived dependencies are illustrated by parallel exact and approximate calculations of the main filtration characteristics. An estimation of the calculated errors due to simplification of the original model was carried out indicating the high accuracy of the approximate solution.**

Keywords—filtration; suspension; nonlinear problem; solution; dependence; method.

I. Introduction

The technological cycle of water purification in accordance with modern standards, as a rule, includes the removal of suspended (filtration) and dissolved (biofiltration) impurities using rapid filters as the most important stage. Thanks to their combination, it is possible to obtain high-quality drinking water enriched with bioadditives. Mass transfer between liquid, solid and biological phases plays a key role in a complex of processes of a physical, chemical and biological nature. It is the stable transfer from the liquid phase to the other two phases that ensures water purification directly and thus the productive operation of the filters. This report focuses on the filtration of aqueous suspensions, which is accompanied by the formation and accumulation of a specific deposit. It is in the deposit that hazardous dispersed impurities are fixed. However, such deposit is often not passive, but actively participates in the absorption of suspended matter. Due to this fact, the formal description of the kinetics of interphase mass transfer is significantly complicated. In general, the kinetic equation is represented in the following form [1, 2]

$$\frac{\partial S}{\partial t} = f_\alpha(S)C - f_\beta(S)\,, \tag{1}$$

where C, S are the (volumetric) concentrations of suspended and deposited suspension particles; f_α and f_β characterize the absorption (adhesion) of suspended and deposited impurity particles. If the deposit is inert with respect to the suspended matter or averaging of the function $f_\alpha(S)$ is allowed and, in addition, proportionality to S is accepted for f_β, then linear filtration takes place. Analytical methods are effective for studying it. A number of exact solutions to linear filtration problems have been obtained [3]. However, as extensive filtration practice shows, linear filtration models often do not provide an adequate pattern. And the main reason for the discrepancy between real and theoretical results is the significant dependence of the rate coefficient of adhesion of dispersed impurities to the packed bed grains on the value S. From a physical point of view, the close connection of f_α with S is due mainly to the limited absorption resource of the filtering material and the autocatalytic effect [4–6]. The specified factors in the case of the presentation

$$f_\alpha(S) = \alpha_0 \chi(S) \tag{2}$$

are taken into account separately or jointly as a first approximation by taking the following expressions for $\chi(S)$

$$\chi_1(S) = S_m - S\,, \tag{3}$$

$$\chi_2(S) = 1 + \theta S\,, \tag{4}$$

$$\chi_3(S) = (S_m - S)(1 + \theta S)\,, \tag{5}$$

where S_m is the dirt-holding capacity relative to the impurity, θ is the autocatalytic coefficient. More complex approximation expressions for $\chi(S)$ allow us to achieve a slightly better agreement between the actual and theoretical results. However, calculations must be performed exclusively by numerical methods. It is realistic to develop engineering methods for calculating nonlinear filtration only based on (3)-(5) and some other representations of a special type.

It was with the aim of creating a theoretical basis for applied calculations of deep bed filtration of a suspension with the active participation of newly formed deposit that its systematic studies were carried out using analytical methods. As a result, it was possible to obtain, firstly, exact solutions to the corresponding nonlinear mathematical problems with a linear function form $\chi(S)$ and a constant filtration rate. Secondly, an approximate solution was found and tested, which, in addition to (3) – (5), also covers a number of other more complex representations for $\chi(S)$.

II. Statement and solution of mathematical problems

The results of filtration studies at $\chi(S)$ according to (2) in the case of an initially contaminated packed bed are presented in [7, 8]. Therefore selective data are provided lower regarding the other two representations. The initial mathematical model includes two connected compartments - clarification and hydrodynamic. The second compartment is not considered here. The first compartment, along with (1), also includes the equation of vertical mass transfer, which can be written after a series of carefully justified simplifications in this way

$$\frac{\partial C}{\partial z}+\frac{\partial S}{\partial t}=0 . \tag{6}$$

The coordinate axis is directed in the direction of movement of the suspension. The system of equations is supplemented with standard boundary and initial conditions

$$z=0,\ C=C_0=const\ ;\quad t=0,\ S=S^0=const\ . \tag{7}$$

An exact solution to the problem (1), (6), (7) for $\chi(S)$ in accordance with (4) was obtained using a number of replacements of the independent and dependent variables, namely,

$$U=\exp\left(\bar{\alpha}_0\bar{z}+\bar{\beta}\,\bar{t}-\bar{\alpha}_0\Phi\right),\ \Phi=\frac{1}{\bar{\alpha}_0}\left(\bar{\alpha}_0\bar{z}+\bar{\beta}\,\bar{t}-\ln U\right).$$

Here and below $\bar{z}=z/L$, $\bar{t}=Vt/(n_0L)$, $\bar{\alpha}_0=L\alpha_0/V$, $\bar{\theta}=n_0C_0\theta$, L is the height of the grain layer, V is the filtration rate, n_0 is the porosity of the clean bed. As a result, for example, for the relative output concentration of suspended matter $\bar{C}_e$, the expression was derived

$$\bar{C}_e\left(\bar{t}\right)= \\ =\frac{I_0\left(2\sqrt{\bar{\alpha}_0\bar{\beta}\bar{t}}\right)-\left(\bar{\alpha}_0\bar{\theta}-\bar{\beta}\right)G_2\left(\bar{z},\bar{t}\right)}{I_0\left(2\sqrt{\bar{\alpha}_0\bar{\beta}\bar{z}\bar{t}}\right)-\left(\bar{\alpha}_0\bar{\theta}-\bar{\beta}\right)G_2\left(\bar{z},\bar{t}\right)+\bar{\alpha}_0G_1\left(\bar{z},\bar{t}\right)} \tag{8}$$

where

$$G_1\left(\bar{z},\bar{t}\right)=\int_0^{\bar{z}}e^{\bar{\alpha}_0\bar{S}_m(1-\bar{S}^0)\xi}I_0\left(2\sqrt{\bar{\beta}\bar{\alpha}_0\bar{S}_m\bar{t}\left(\bar{z}-\xi\right)}\right)d\xi.$$

$$G_2\left(\bar{z},\bar{t}\right)=\int_0^{\bar{t}}e^{-\left(\bar{\alpha}_0\bar{\theta}-\bar{\beta}\right)\left(\bar{t}-\xi\right)}I_0\left(2\sqrt{\bar{\alpha}_0\bar{\beta}\,\bar{z}\xi}\right)d\xi\ ,$$

I_0 is the symbol of the Bessel function of the imaginary argument of the first kind of zero order.

To illustrate this solution, the $\bar{C}_e$ concentration was determined using formula (8) with the following typical initial data: $\bar{\alpha}_0=5,\ \bar{\beta}=0.01$. The curves of the growth of the specified concentration over time, calculated using the software package Mathcad, are shown in Fig. 1. It clearly demonstrates the significance of the coefficient $\bar{\theta}$ for long-term clarification of the suspension. The calculated curves corresponding to $\bar{\theta}>0$, noticeably deviate downward from the base curve for comparison, curve 1 ($\bar{\theta}=0$), starting approximately from the moment $\bar{t}=70$.

If the adhesion function is nonlinear, then exact methods are ineffective. Then it is possible to derive calculated dependencies only using approximate methods. In this case, estimation of their accuracy becomes of great importance. It is within the framework of this topic that special simplification techniques have been developed that take into account the specific features of the behavior of the filteration characteristics and do not allow any linearization of the original nonlinear model. The essence of these techniques is the dynamic averaging of the concentration C in the kinetic equation (1).

After a series of calculations of varying complexity, the approximate solution was obtained in parametric form, where the parameter is the value $\bar{C}_a$, specially introduced during the solution of the problem. Thus, to carry out specific calculations, it is necessary to involve two dependencies connecting $\bar{C}$ and $\bar{t}$ with $\bar{C}_a$. It was received

$$\bar{C}(\bar{C}_a,\bar{t})=\frac{C}{C_0}=e^{-\mu(\bar{C}_a,\,\bar{t})}\times$$

$$\times\left[1+\bar{\beta}\bar{t}\int_1^{\bar{C}_a}\frac{\bar{S}(\zeta,\bar{t})}{\left[\bar{S}^0-\sigma_1(\zeta)\right]\left(1-e^{-(\bar{\alpha}_0\zeta+\bar{\beta})\bar{t}}\right)}e^{\mu(\zeta,\,\bar{t})}d\zeta\right] \tag{9}$$

$$\bar{z}=\bar{t}\int_1^{\bar{C}_a}\frac{d\zeta}{\left[\bar{S}^0-\sigma(\zeta)\right]\left(1-e^{-(\bar{\alpha}_0\zeta+\bar{\beta})\bar{t}}\right)}, \tag{10}$$

where $\mu(\bar{C}_a,\bar{t})=\bar{\alpha}_0T\int_{\bar{C}_a}^{1}\frac{\bar{S}_m-\bar{S}(\zeta,\bar{t})}{\bar{S}^0-\sigma(\zeta)}\frac{d\zeta}{1-e^{-(\bar{\alpha}_0\zeta+\bar{\beta})\bar{t}}}$,

$\sigma(\bar{C}_a)=\bar{\alpha}_0\bar{S}_m\bar{C}_a/(\bar{\alpha}_0\bar{C}_a+\bar{\beta})$.

III. Discussion of approximate results

Approximate calculations were performed for two series of examples. The first series was intended precisely to evaluate the accuracy of the $\bar{C}$ calculations and, in general, the approximate solution. Its high accuracy is evidenced by fig. 2. Calculations were carried out approximately according to (9), (10) and exactly using the corresponding formula from [7]. The following characteristic values of model coefficients were used as initial data: $\bar{\beta}=0.005$, $\bar{S}_m=5000$, $\bar{\alpha}_0=0.001$, 0.0015. Calculation errors were significantly larger at $\bar{\alpha}_0=0.0015$ reaching a maximum value of 9.5%. However, with continued filtration ($\bar{t}>500$) they noticeably decreased. If the absorptive capacity of the bed was one and a half times weaker, then the maximum error did not exceed 2% and both calculated curves (1, 2) almost merged during the calculation period.

Also, the subject of approximate calculations was the concentration profiles of the deposited particles and the increase in head losses over time for linear (3), (4) and nonlinear (5) forms of $\chi(S)$. In conclusion, the reaction of technological times (protective action, achievement of maximum permissible head losses and filter run duration) to hypothetical changes in the absorption capacity of the filter material and deposit was analyzed using exact and approximate dependencies. In general, the results of a detailed quantitative analysis demonstrated the high capabilities and reliability of the new method for calculating nonlinear filtration of aqueous suspensions.

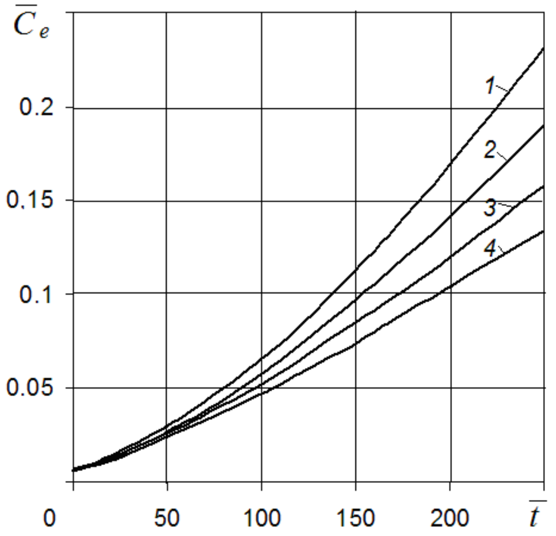

Figure 1. Increase in the relative concentration of suspended matter in the filtrate:

1 – $\overline{\theta} = 0$, *2 –* $\overline{\theta} = 0.0005$,

3 – $\overline{\theta} = 0.001$, *4 –* $\overline{\theta} = 0.0015$

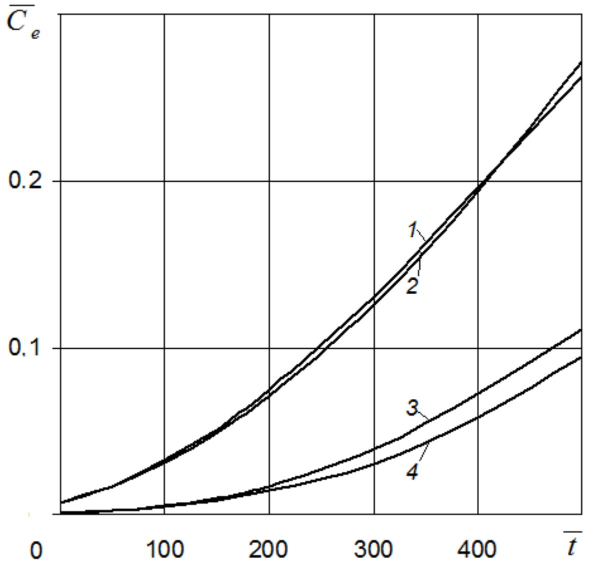

Figure 2. Increase in the relative concentration of suspended matter in the filtrate over time:

1,3 - approximate calculation;

2,4 - accurate; 1,2 - $\overline{\alpha}_0 = 0,001$;

3,4 - $\overline{\alpha}_0 = 0,0015$

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