

# On the problems of determining the trajectory of an aerial vehicle based on inertial navigation system data

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**Abstract**—Navigation in military UAVs requires the use of onboard inertial systems. An approach is proposed for noise filtering, refinement of initial and intermediate values when applying calculation algorithms based on the estimates of loads and angular velocity taken from sensors.

**Keywords**—inertial navigation system; accelerometer; gyroscope; orientation angles; aerial vehicle; trajectory.

## I. INTRODUCTION

The introduction of advanced technologies in electronics and materials science has led to the development of various types of UAVs and their widespread use in the military. Civilian applications of UAVs require stable communication. Satellite navigation systems such as GPS, GLONASS and others can also be used to determine the current location. In military applications, the limitations of communication and satellite navigation systems become apparent. Therefore, one of the main problems faced by UAV control system developers is the problem of UAV self-location in the absence of a GPS system. As is known, one way to solve problems of this type is by processing the inertial navigation data of the aerial vehicle. This approach involves equipping the vehicle with an accelerometer and gyroscope system. When classical inertial navigation systems are used, the initial position is input by the operator (pilot), the accelerometer measures loads on the ground or on the aerial vehicle (vector components expressing the ratio of the force acting on the vehicle to its weight), and the gyroscope measures rotation angles (e.g., [1, 2]). For both civilian and military aircraft, orientation-related information is entered into the system at the apron where it is parked before takeoff.

For the purpose of this paper, a UAV is defined as an airplane-type fixed-wing unmanned aerial vehicle.

Today, unmanned aerial vehicles are launched from pads that undergo no special preparation. One of the most common navigation sensors used in UAVs is MPU6000 [3]. Navigation sensors used in some cases measure not orientation angles, but their rate of change (e.g., MPU6050 [4]). Sensors of this type also have constant measurement data errors. Such sensor errors present no particular problems in the visual control of commonly used short-range quadcopters or hexacopters. UAVs of this type are also equipped with magnetometers for determining the directions of the horizon.

The above characteristics require an adequate data processing algorithm to control a relatively long-range (40÷50 km) UAV with wing dimensions within 3.0÷5.0 m.

The paper describes the proposed approach to finding the values used to calculate the flight path of a winged aerial vehicle equipped with a magnetometer and a MPU6000-type sensor.

Note that when analyzing a flight within a range of 50-100 km, the curvature of the Earth can be ignored, so the mathematical apparatus below is given for rectangular coordinate systems.

## II. COORDINATE SYSTEMS USED, DESCRIPTION OF QUANTITIES

It is convenient to use a UAV-fixed local coordinate system  $Oxyz$  (Fig.1) to analyze the forces and moments acting during flight simulation, and an Earth-fixed rectangular coordinate system  $O_gXYZ$  (Fig.2) to describe the trajectory. We introduce the respective coordinate systems.

We will assume that in the UAV-fixed coordinate system, the origin of coordinates  $O$  is at the center of gravity of the vehicle, with the axis  $Ox$  oriented along the fuselage of the vehicle and the axis  $Oz$  oriented

perpendicular to it in the direction of the right wing. The axis  $Oy$  is perpendicular to the plane  $Oxz$  so that the coordinate system  $Oxyz$  is positively oriented.

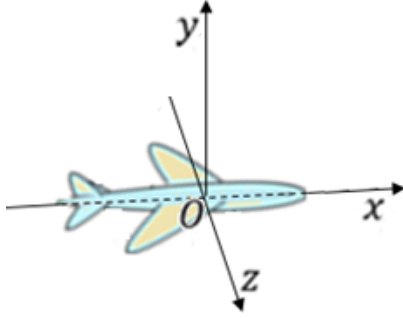


Figure 1. UAV-fixed coordinate system.

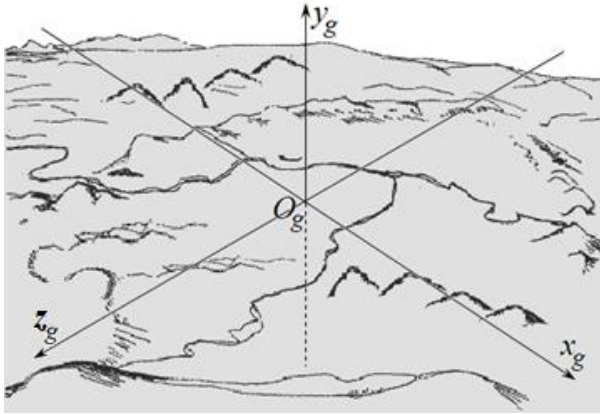


Figure 2. Earth-fixed coordinate system.

For simplicity, the origin of the Earth-fixed rectangular coordinate system  $O_gXYZ$  is identified with the starting point of the UAV trajectory, with the axis  $O_gX$  oriented upward perpendicular to the Earth's surface. We point the axes  $O_gX$  and  $O_gZ$  so that  $O_gXYZ$  is positively oriented. For definiteness, we assume that the axis  $O_gX$  is oriented northward.

The coordinates of the UAV during flight are set relative to the system  $O_gXYZ$ .

The spatial position of the UAV is determined by the position of the coordinate system  $Oxyz$  relative to the system  $O_gXYZ$  (Fig.3). The orientation angles  $\psi, \theta, \gamma$  are used for this purpose.

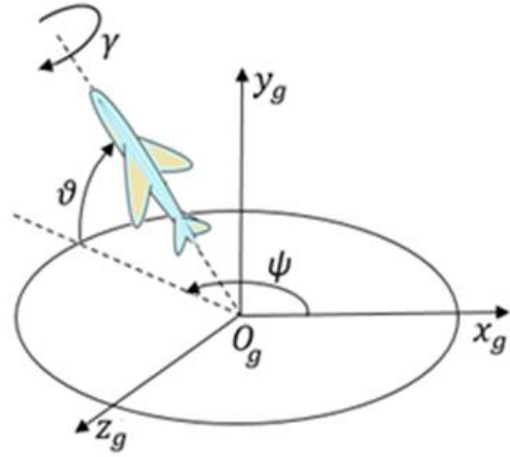


Figure 3. Orientation (Krylov) angles.

$\psi$  is the yaw angle – the angle between the axis  $O_gX$  of the coordinate system  $O_gXYZ$  and the projection of the axis  $Ox$  on the plane  $O_gXY$ ;  $\theta$  is the pitch angle – the angle between the axis  $Ox$  and the plane  $O_gXZ$ ;  $\gamma$  is the roll angle – the angle between the axis  $Oz$  and the axis  $O_gZ$  [5].

Note that the following matrix  $A$  can be used to calculate the coordinates of a vector given in the system  $O_gXYZ$  relative to the system  $Oxyz$  [6]:

$$A = \begin{bmatrix} C_\theta C_\psi & S_\theta & -C_\theta S_\psi \\ -C_\gamma S_\theta C_\psi + S_\gamma S_\psi & C_\gamma C_\theta & C_\gamma S_\theta S_\psi + S_\gamma C_\psi \\ S_\gamma S_\theta C_\psi + C_\gamma S_\psi & -S_\gamma C_\theta & -S_\gamma S_\theta S_\psi + C_\gamma C_\psi \end{bmatrix},$$

where for simplicity  $C_\xi \equiv \text{Cos}\xi$ ,  $S_\xi \equiv \text{Sin}\xi$ .

### III. INITIAL PROCESSING OF SENSOR DATA

Data from MPU6000 sensors, designed for a wide range of applications, can be retrieved in various frequency ranges (1÷100 Hz). As a rule, depending on the inertia characteristics of the object of study, the optimal frequency of data retrieval required is determined at the stage of UAV design. The choice of high frequency increases the time of data processing, overloading computing resources. On the other hand, if a low frequency is chosen, data received may be insufficient to calculate the trajectory with the required accuracy.

It should also be noted that loads and angular velocities measured along different axes are measured successively, but are referenced to the same time instant. Therefore, the values of these quantities transmitted by the sensor to the system at a given instant may differ slightly from the real values.

Fig. 4 and 5 show examples of sensor data collected for a UAV at rest. Analysis of these data shows that even in the absence of motion, if the loads vary in a range close to the real values, the angular velocities vary in a range of non-zero values. Thus, when using MPU6000 sensors for flight data retrieval, the *problem of optimal frequency selection and initial data processing* arises.

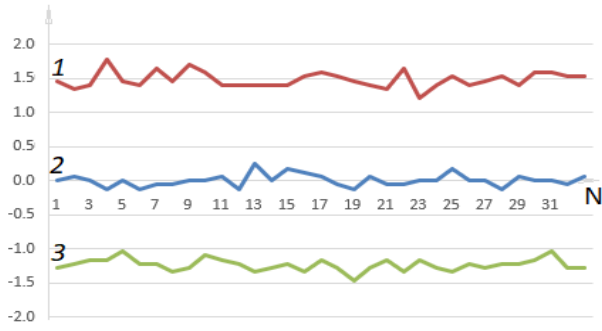


Figure 4. Graph of pitch (1), yaw (2) and roll (3) angular velocities at rest vs. time, with the time instants (s) when measurements are taken indicated along the horizontal axis, and angular velocities along the vertical axis (degree/s).

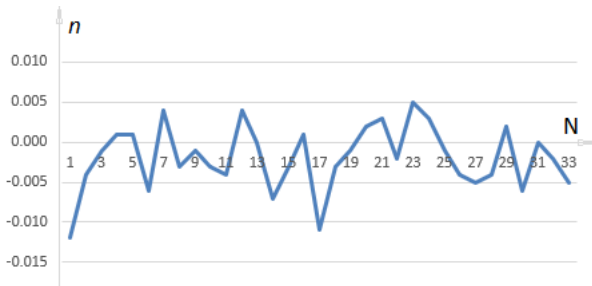


Figure 5. Graph of variation of the load on the axis  $Ox$  at rest vs. time, with the time instants (s) when measurements are taken indicated along the horizontal axis, the value of the load along the vertical axis.

Based on numerous experiments conducted with the inertial characteristics of medium-sized winged UAVs, the choice of a measurement frequency of 10 Hz was deemed satisfactory. On the other hand, it is sufficient that the system of differential equations describing the trajectory be solved at 1-second increments.

A procedure similar to the practice of entering aircraft initial data into the navigation system at the preflight apron is proposed. For example, an UAV mounted on the pad and preparing for flight does not move for the first 10÷15 seconds. We denote this period by  $T$ . Suppose that the measurements were taken at the time instants  $t_i = i \cdot \Delta t, i = 0, 1, \dots, N, N = [T/\Delta t]$ .

Accordingly, we take  $\Delta t = 0.1 \text{ s}$ . Denote the angular velocities received from the MPU6000 sensor during this time by  $\psi'_i, \theta'_i, \gamma'_i$ . Then the corresponding average sensor values in the period  $T$  for these quantities can be calculated as follows:

$$\begin{cases} \psi'_* = \frac{1}{N} \sum_{i=0}^N \psi'_i, \\ \theta'_* = \frac{1}{N} \sum_{i=0}^N \theta'_i, \\ \gamma'_* = \frac{1}{N} \sum_{i=0}^N \gamma'_i. \end{cases}$$

Since the values  $\psi'_*, \theta'_*, \gamma'_*$  are the preflight sensor values of the quantities under consideration, it is obvious that they must correspond to the real zero

values. Therefore, the values of angular velocities for the subsequent period are reduced to the values  $\psi'_*, \theta'_*, \gamma'_*$ .

To denote the values of angular velocities and loads measured at the time instants  $t_{j-1} + 0.1 \times k$  at a frequency of 10 Hz in the interval  $[j-1, j]$ , we introduce the index  $\{j\}_k, (k = 1, 2, \dots, 10)$ . The following formulas (filters) are proposed to calculate the value of these quantities:

$$\begin{cases} \tilde{\psi}'_j = 0.1 \cdot \sum_{k=0}^9 \psi'_{\{j\}_k} - \psi'_*, \\ \tilde{\theta}'_j = 0.1 \cdot \sum_{k=0}^9 \theta'_{\{j\}_k} - \theta'_*, \\ \tilde{\gamma}'_j = 0.1 \cdot \sum_{k=0}^9 \gamma'_{\{j\}_k} - \gamma'_*, \\ \tilde{n}_{x,j} = 0.1 \cdot \sum_{k=0}^9 n_{x,\{j\}_k}, \\ \tilde{n}_{y,j} = 0.1 \cdot \sum_{k=0}^9 n_{y,\{j\}_k}, \\ \tilde{n}_{z,j} = 0.1 \cdot \sum_{k=0}^9 n_{z,\{j\}_k}. \end{cases}$$

#### IV. CALCULATING THE INITIAL ORIENTATION

The UAV trajectory is calculated by integrating its motion equations, and the correct calculation of this trajectory depends on the initial position. While incorrect determination of the initial coordinates causes a shift of the calculated UAV trajectory parallel to the real trajectory, incorrect assignment of the initial values of the orientation angles expressing the initial position can significantly distort the trajectory.

Since the gyroscope of an MPU6000 sensor calculates the rate of change of orientation angles, there is a need to use data from other sources to calculate these angles at the time instant  $t = 0$ . Thus, when using MPU6000 sensors, **the problem of determining the initial values of the orientation angles**  $\varphi, \theta, \gamma$  arises.

In this study, the initial value of the yaw angle  $\psi_0$  is calculated on the basis of magnetometer data. The following approach is proposed to calculate the pitch and roll angles.

Introduce

$$\begin{cases} a_{x,0} = g \cdot \tilde{n}_{x,11}, \\ a_{y,0} = g \cdot \tilde{n}_{y,11}, \\ a_{z,0} = g \cdot \tilde{n}_{z,11}, \end{cases} \quad (1)$$

where  $g = 9.81 \text{ m/s}^2$ . As mentioned above, the UAV is assumed to be stationary for the first 10÷15 seconds. Using the matrix  $\mathbf{A}$  introduced above, we can write the components of the gravitational acceleration that affect the UAV during this time for the coordinate system  $Oxyz$  as follows:

$$A \cdot \begin{pmatrix} 0 \\ -g \\ 0 \end{pmatrix} = \begin{pmatrix} a_{x,0} \\ a_{y,0} \\ a_{z,0} \end{pmatrix}. \quad (2)$$

Then we get the following formulas to calculate the values of the orientation angles  $\theta, \gamma$  corresponding to the instant  $t = 0$ :

$$\begin{cases} \theta_0 = \arcsin \frac{a_{x,0}}{g}, \\ \gamma_0 = -\arctg \frac{a_{z,0}}{a_{y,0}}. \end{cases} \quad (3)$$

## V. CALCULATING THE ACCELERATION

Accumulation of angular velocity measurement errors by the MPU6000 sensor during long flights causes the calculated trajectory to deviate significantly from the actual flight path. Therefore, the **problem of periodic refinement of orientation angles** on the basis of other data during the flight arises.

The following approach is proposed in this study to solve this problem.

We assume that a UAV flying in a stable engine operation mode moves in a straight line at a constant speed, keeping its orientation in some parts of the trajectory unchanged. This period of time can last up to 5÷6 seconds. We denote the UAV orientation angles for this period of time by  $\psi_0, \theta_0, \gamma_0$ . Given that this is a relatively robust sensor, the angle  $\psi_0$  can still be determined using the magnetometer.

Suppose that the time instants  $t_i, t_{i+1}, \dots, t_{i+N-1}$ , where  $N \cong 55$ , belong to the period of stable flight of the UAV. Using formulas (1), based on the load data coming from the sensors at the same time instants, we can calculate the accelerations  $a_{x,i+j}, a_{y,i+j}, a_{z,i+j}$ ,  $j = 0, 1, \dots, N-1$ .

One of the main indications of rectilinear flight at a constant speed is the equality

$$a_{x,i+j}^2 + a_{y,i+j}^2 + a_{z,i+j}^2 = g^2 \quad (4)$$

being satisfied for all  $j$ . Given relations (2), the following equalities have to be satisfied in the ideal case as well:

$$\begin{cases} a_{x,i+j} = g \sin \theta_0, \\ a_{y,i+j} = g \cos \gamma_0 \cos \theta_0, \\ a_{z,i+j} = -g \sin \gamma_0 \cos \theta_0. \end{cases} \quad (5)$$

But due to sensor measurement errors, equalities (4) and (5) are not satisfied with sufficient accuracy.

We denote the accelerations determined by the right-hand side of equations (5) by  $\tilde{a}_{x,0}, \tilde{a}_{y,0}, \tilde{a}_{z,0}$ , respectively. Then the condition that these equations are satisfied on average with accuracy  $\varepsilon$  can be written as follows:

$$J(\tilde{a}_{x,0}, \tilde{a}_{y,0}, \tilde{a}_{z,0}) \equiv \sum_{j=0}^{N-1} \left[ (a_{x,i+j} - \tilde{a}_{x,0})^2 + (a_{y,i+j} - \tilde{a}_{y,0})^2 + (a_{z,i+j} - \tilde{a}_{z,0})^2 \right] \leq \varepsilon^2, \quad (6)$$

where  $\varepsilon$  is a known number calculated within the measurement error of the MPU6000 sensor.

From the minimality condition of the function  $J(\tilde{a}_{x,0}, \tilde{a}_{y,0}, \tilde{a}_{z,0})$  we can obtain the following possible values for the quantities  $\tilde{a}_{x,0}, \tilde{a}_{y,0}, \tilde{a}_{z,0}$ :

$$\begin{cases} \tilde{a}_{x,0} = \frac{1}{N} \sum_{j=0}^{N-1} a_{x,i+j}, \\ \tilde{a}_{y,0} = \frac{1}{N} \sum_{j=0}^{N-1} a_{y,i+j}, \\ \tilde{a}_{z,0} = \frac{1}{N} \sum_{j=0}^{N-1} a_{z,i+j}. \end{cases} \quad (7)$$

Thus, the following algorithm can be used to solve the problem of periodic refinement of the orientation angles.

- Verify that the accelerations  $a_{x,i+j}, a_{y,i+j}, a_{z,i+j}$ ,  $(j = 0, 1, \dots, N-1)$  calculated from the loads measured at the time instants  $t = t_{i+j}$  satisfy necessary conditions (4) for the current instant  $t_{i+N-1}$  with sufficient accuracy.
- If conditions (4) are satisfied, calculate the accelerations  $\tilde{a}_{x,0}, \tilde{a}_{y,0}, \tilde{a}_{z,0}$  using formulas (7).
- Verify that inequality (6) is satisfied for the calculated accelerations.
- If (6) is satisfied, it is considered that the UAV is moving in a straight line at a constant speed, and its orientation angles  $\theta_0, \gamma_0$  are calculated from formulas (3).

## VI. CONCLUSION

We demonstrate the possibility of using the data from inertial sensors to calculate the trajectory of unmanned aerial vehicles. Since sensor data are noisy, a filtering algorithm is applied. The orientation angles-related data of the MPU6050 sensor is the value of angular velocity. The initial position being known is an important condition for calculating the orientation angles. The paper describes the procedure for finding the angles at the initial position. Furthermore, an algorithm is presented for refining the values of the orientation angles using loads on the section of the trajectory where the UAV flies in a straight line at a constant speed. The procedure for using the refined data from the sensors in the calculation of the trajectory is given.

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