

# On optimal sparse control formulation for reconstruction of noise-affected images

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**Abstract** – We develop a novel approach to the domain decomposition basing on the novel anisotropic version of the Chan-Vese active contour model for a special domain decomposition problem which is motivated by huge number of different applications especially the satellite image segmentation under the remote sensing of the Earth surface.

**Keywords** – component, formatting, style, styling, insert.

We discuss a new variational problem which is suggested by applications to satellite image segmentation. Since the satellite images are an important source for extracting landscape boundaries and other vegetation structures which can provided extremely useful insights for applications in environmental monitoring, agriculture, forestry, and other related fields. In particular, in agricultural crop field classification, one of a fundamental problem is to provide a disjunctive decomposition of a fixed domain onto finite number of nonempty subsets such that each of these subsets could be associated with a crop that is grown in this area, or with a forest regions, or water zones, and so on, and this correspondence must be established at rather high level of accuracy. Up-to-date and accurate crop maps (or crop field classification) are needed to update agricultural statistics, to provide agricultural crop yield prediction, and are often used in environmental modeling. Typically, such association between a given region and some agricultural crop can be made through the detection and quantitative assessment of green vegetation which is one of the major application of remote sensing studies.

Thus, each pixel of the original image can be associated with the corresponding VI-feature. The problem, which is suggested by application to remote sensing satellite image processing, consists in computing a decomposition of the domain  $\Omega$  of a given image such that

- the VI-characteristic varies smoothly and/or slowly within each subset of  $\Omega$ ;
- the VI-characteristic varies discontinuously and/or rapidly across most of the boundary between different subsets.

As distinguished features of this statement is that we cannot reduce it to the standard settings of segmentation

problem (see, for instance, the Mumford-Shah energy based model and others) are the following ones:

- Each region of  $\Omega$  should consist of pixels that can be reasonably grouped according to the VI-characteristic. Simultaneously, these regions should be easy to differentiate according to the chosen image feature;
- The respective interiors of image regions should have a more or less simple geometry without gaps. Boundaries of image regions should be smooth enough but also accurate with respect to the chosen image feature;
- The most restrictive obstacle in the construction of such decomposition is the fact that these subdomains should not overlap the borders between fields or contain any fragments of such borders, meaning that they cannot take in even small parts of different fields with arguably different crops.

All of these makes the above mentioned segmentation problem rather challenging. It is enough to observe that a precise consideration of this problem demonstrates that the quantitative interpretation of remote sensing information from vegetation is a complex task. Many studies have limited this interpretation by assumption that the extracting vegetation information uniformly and smoothly distributed within the particular crop fields. However, this assumption is evidently broken when trying to apply these type vegetation indices on heterogeneous canopies such as plantations with a mixed combination of soil, weeds, and other crops, or plantation where the vegetation of interest has different VI-characteristic due to spacial variability.

The main idea, we realize in the new setting of variational problem, can be briefly described as follows. We propose to make use of the so-called  $f$ -decomposition instead the standard Chan-Vese 'active contours without edges' model. The role of the function  $f$  in such decomposition has to guarantee that the new objects after the  $f$ -decomposition will have homogeneous values of the target function  $f$  withig each separate field. Thus, we formulate the segmentation problem as a constrained minimization problem in a special anisotropic BV space, where the 'effect of anisotropy' we associate with the structure topology of VI-distribution.