Modeling of Filtration Processes with Extended Sources and Homogenization

https://doi.org/10.31713/MCIT.2024.082

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Abstract – **Methods for modeling dynamic filtration processes with extended sources will be considered. These methods are based on the homogenization of the relevant initial boundary value problems. This approach leads to the description of several filtration modes, the implementation of which depends on the orders of the filtration, transfer, and potentiality coefficients. The problems for describing these modes have constant coefficients and can be solved analytically or numerically. The accuracy of the approximations is also established.**

Keywords – **initial boundary value problems; asymptotic expansion; accuracy estimates; multiscale porous media; approximation.**

I. Introduction

Modern wells are realized with extended horizontal drains and sources, which are the main elements in oil and gas production. It is an interesting mystery how it is possible to realize such wells, which usually start vertically and then turn into a horizontal plane (sometimes such horizontal components are realized in several parallel planes, which is an even bigger mystery). But it is generally accepted that such wells are the most effective as drains and sources. Sometimes these sources are used to pump auxiliary fluids, which are often heated to better affect pumping out through drains from the well.

Such wells can be used to solve environmental problems, for example, to prevent groundwater pollution and to find methods for cleaning already polluted water resources. From an environmental point of view, it is also important to understand what will remain after using such wells for oil and gas production. Studying these processes using engineering observation methods in underground environments seems to be a rather expensive undertaking, since it requires the installation of various sensors at many points and at different depths. In addition, such processes are dynamic and long-lasting, and therefore require long-term monitoring of sensors, which can also be very expensive.

Thus, mathematical modeling is relevant for studying and understanding these processes, since the availability of sufficiently powerful computers and modern software packages allows for fairly quick calculations, for example, of possible modes of oil and gas production or groundwater purification. However, before implementing the calculations, it is necessary to select a model and conduct a mathematical analysis of this model. The real model is a dynamic model of (single-phase) filtration in a porous medium, where the presence of extended horizontal drains and sources is taken into account by suitable delta functions.

In this case, drains and sources differ only in the sign of these delta functions, and then only sources can be considered. In addition, the equations of this model are linear, so only one delta function can be used to model a horizontal source. These are the models that will be discussed here. However, it should be taken into account that delta functions are quite singular and standard methods are not directly applicable. In the future, it is not difficult to transfer the methods used here to two-phase filtration and to coupled models that take into account heat distribution when heating injected liquids.

The coupled models of the process of two-phase non-isothermal filtration are considered in sufficient detail, for example, in [1], where methods of complex analysis are used, which are essentially two-dimensional and are applicable only to domains located on a plane. The stationary single-phase version of this model is considered in [2], where three possible filtration regimes are described by homogenization methods.

These regimes depend on the orders of the filtration coefficients in comparison with the microscale coefficient. This scale coefficient arises from the assumption that the porous medium is heterogeneous and multiscale. A periodic medium is chosen as the simplest model there, which made it possible to apply asymptotic methods for homogenizing heterogeneity and separating the scales of the problem using the asymptotic expansion methods presented in detail in [3]. Periodicity is natural for artificial porous media. However, promisingly, the homogenization methods used are quite standardly generalized to stochastic multiscale media. Note that the multi-scale nature is typical for the problems under consideration, for example, according to [4], where different versions of these problems are presented and discussed.

II. Methods

It is known [5] that usually in filtration models homogeneous media are considered, the filtration coefficients of which are obtained by volume averaging without justification of this transition. However, with this approach it is not clear how to distinguish filtration modes by separating scales, which in a sense are erased by volume averaging. Thus, we consider a periodic porous medium with sources, which is modeled by initial boundary value problems for non-stationary equations depending on additional multiscale parameters. Homogenization of such problems with additional parameters for non-singular data is discussed in some detail in [3].

This approach, presented also in [2], makes it possible to separate scales and identify possible filtration modes for singular data. This allows us to obtain homogenized initial boundary value problems as approximations of the solutions under study and to prove approximation accuracy. Another approach is known, using which multiscale problems are get as approximations for solutions of filtration problems in periodic porous media, for example, according to [6] and [7]. But, the two-scale homogenized problems depend on two micro and macro variables and the type of such equations is not clear. Moreover, in this case, the accuracy approximation is not proven.

Thus, dynamic filtration processes with extended sources in periodic media and homogenization will be discussed. The resulting homogenized models will depend on the orders of the filtration coefficients and potentiality. These coefficients are variable, but naturally can also be constant. However, the emergence of possible filtering modes is influenced primarily by the orders of the coefficients in comparison with the scale parameter. To derive these models, the asymptotic scale separation algorithm presented in [3] is used substantially. This makes it possible to separate scale variables and describe homogenized filtering modes. The sources under consideration are modeled by delta functions, which are quite singular and the results of [8] and [9] will be useful here.

Stationary versions of the discussed problems are also used as models for simulating blood flows, vascular networks and transport processes in tissue, for example, according to [10]–[14]. The weighted space method was used in [10] and the singularity subtraction method was applied in [11] and [12] to study such problems. General methods for simulating such problems were proposed in [13] and [14]. Such methods were applied to equations with constant coefficients and it is not clear whether they can be generalized to non-stationary equations with variable coefficients. The methods of a priori estimates and singularity subtractions are used here.

III. Conclusions

Thus, homogenization methods for modeling dynamic filtration processes with extended sources are discussed. These methods lead to approximations of solutions to the dynamic problems under consideration. Estimates of the accuracy of such approximations are also discussed. Depending on the orders of the coefficients of filtration, transport, and potentiality compared to the multiscale parameter, several filtration regimes are investigated. Understanding such regimes may be useful for broader applications beyond filtration, as these patterns may arise in simulating blood flows, vascular networks, and transport in tissue processes.

Acknowledgment

This work has been supported by Grant of the Ministry of Education and Science of Ukraine for perspective development of a scientific direction ”Mathematical sciences and natural sciences” at Taras Shevchenko National University of Kyiv.

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