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Modification of the filtration consolidation equation and kinematic boundary condition into the case of biodegradation processes

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Abstract – **This work considers the modification of the filtration consolidation equation to describe the settlement processes of municipal solid waste (MSW) landfills, taking into account the influence of biodegradation processes. The mathematical model incorporates the thermal effect resulting from biodegradation processes and introduces boundary conditions to describe the interaction of heat and fluid pressure. The corresponding mathematical model is described by the Stefan boundary value problem for a system of differential equations of biodegradation and two quasilinear parabolic equations. One of the equations describes the change in thermal state in a porous medium, while the other describes the change in excess pore pressure in the pore fluid. This approach allows for avoiding the determination of the stress-strain state and modeling the microbiological dynamics. The finite element method was used to find the solutions to the corresponding boundary value problem. The obtained results assess the change in the dynamics of the settlement of MSW landfills considering biodegradation and non-isothermal conditions**

Keywords – MSW; settlement; biodegradation; consolidation; boundary condition; thermal processes

I. Introduction

The disposal of MSW through the creation of special landfills is an important environmental issue. As noted in [1], according to World Bank statistics, the global amount of generated MSW was 2.01 billion tons in 2016, and this figure is projected to reach 3.4 billion tons by 2050. The composition of this waste depends on landfill management methods. One of the main problems is the settlement of landfills. Predicting this process requires consideration of temperature regime, humidity, and biodegradation processes [2].

To model the settlement process, theoretical analysis, laboratory studies, numerical modeling, and field measurements are employed. Each of these approaches has its advantages and disadvantages. For example, models that account for biodegradation often assume that the pore space created by the decomposition of organic residues is fully filled with solid phase. However, such models frequently depend on the kinetics of biodegradation processes and field experiments, complicating their use for modeling. The paper [3] examined the influence of microorganism dynamics on the consolidation of a waste repository. However, it did not address the issues of temperature effects and greenhouse gas generation.

One example of previous research is the work in [4], which developed a theory of one-dimensional nonlinear consolidation considering the thermal effect. However, this approach did not take into account the biodegradation processes, which are key factors in landfill settlement. This process is a result of biodegradation processes and mechanical compaction of materials. Biodegradation processes are accompanied by the release of greenhouse gases and heat, which complicates the prediction of landfill settlement dynamics. Temperature effects have an important impact on the environment and on processes in porous media. For example, the influence of non-isothermal conditions on humus formation processes was studied in [5].

This paper presents a modification of the filtration consolidation model that accounts for biodegradation processes. This approach allows for the description of settlement processes through a system of equations for the kinetics of biodegradation and two quasilinear equations that describe changes in thermal state and pore pressure. The application of this approach enables the avoidance of modeling microbiological dynamics and the stress-strain state, which in turn simplifies the model.

II. Kinematic boundary condition for the upper movable surface of the landfill considering biodegradation processes

The settlement process of the surface of MSW landfills is largely determined by the biodegradation of organic waste residues. To model this process, a kinematic boundary condition has been developed that accounts for the movement of the upper boundary of the landfill due to biodegradation. In this model, settlement depends on the modification of the volume of the porous medium, which is influenced by the amount of organic residue transformation and changes in pore space.

In this model, as in the work [6], we assume that the surface settlement occurs in the vertical direction. To maintain the consistency of the material presentation, we will present some already known facts and derivations. First

$$\frac{dl(t)}{dt} = -\int_{l(t)}^{L} \frac{d\varepsilon(z,t)}{dt} dz, \quad (1)$$

where the integration is carried out over the vertical segment $[l(t);\, L]$, $l(t)$ is the upper boundary of the porous medium, L is the lower boundary of the porous medium, and $\varepsilon(x,t)$ is the volumetric strain of the porous medium. Secondly, it is known that condition (1) was derived based on the integral sum

$$\frac{l(t+\Delta t) - l(t)}{\Delta t} = -\sum_{i=1}^{m} \frac{\varepsilon_i(t+\Delta t) - \varepsilon_i(t)}{\Delta t} \Delta z_i,$$

where m is the number of fragments with volumes $V_i (i = \overline{1,m})$ of the porous medium, covering the vertical segment $[l(t);\, L]$.

Considering the biodegradation processes, the volume V_i change over a certain time interval Δt can occur due to changes in pore volume $\Pi_i V_i$ as well as changes in the volume of organic components of the porous medium, $W_i V_i$. Therefore, we obtain

$$\begin{aligned} V_i(t) - V_i(t+\Delta t) = \Pi_i(t) V_i(t) - \Pi_i(t+\Delta t) \times \\ \times V_i(t+\Delta t) + W_i(t) V_i(t) - W_i(t+\Delta t) V_i(t+\Delta t), \end{aligned} \quad (2)$$

where $\Pi_i(t)$ is the average porosity of the medium in the selected segment, $W_i(t)$ is the relative volumetric content of the organic component in the selected segment (both quantities are dimensionless, expressed in $\frac{m^3}{m^3}$).

Considering that $\Delta V_i = V_i(t) - V_i(t + \Delta t)$ from equation (2), we obtain

$$\frac{\Delta V_i}{V_i(t)} = \Pi_i(t) - \Pi_i(t+\Delta t) + \Pi_i(t+\Delta t) \frac{\Delta V_i}{V_i(t)} +$$

$$+ W_i(t) - W_i(t+\Delta t) + W_i(t+\Delta t) \frac{\Delta V_i}{V_i(t)}.$$

Since, by definition, the relative volumetric strain of the porous medium is $\frac{\Delta V_i}{V_i(t)} = \varepsilon_i$, then

$$\Delta\varepsilon_i = \Pi_i(t) - \Pi_i(t+\Delta t) + \Pi_i(t+\Delta t)\Delta\varepsilon_i +$$

$$+ W_i(t) - W_i(t+\Delta t) + W_i(t+\Delta t)\Delta\varepsilon_i,$$

$$\left(1 - \Pi_i(t+\Delta t) - W_i(t+\Delta t)\right)\Delta\varepsilon_i =$$

$$= -\left(\frac{d\Pi_i(t)}{dt} + \frac{dW_i(t)}{dt}\right)\Delta t,$$

where in the above relation, higher-order infinitesimals are neglected (as $\Delta t \to 0$).

Taking into account that $\Pi_i = \frac{e_i}{1+e_i}$,where e_i represents the average void ratio in the chosen segment of the medium, we derive

$$\left(1 - \frac{e_i(t+\Delta t)}{1 + e_i(t+\Delta t)} - W_i(t+\Delta t)\right)\Delta\varepsilon_i =$$

$$= -\frac{d}{dt}\left(\frac{e_i(t)}{1 + e_i(t)}\right)\Delta t - \frac{dW_i(t)}{dt}\Delta t,$$

$$\frac{\Delta\varepsilon_i}{\Delta t} = -\frac{1}{1 - \left(1 + e_i(t+\Delta t)\right)W_i(t+\Delta t)} \times$$

$$\times\left(\frac{1 + e_i(t+\Delta t)}{\left(1 + e_i(t)\right)^2}\frac{de_i(t)}{dt} + \left(1 + e_i(t+\Delta t)\right)\frac{dW_i(t)}{dt}\right),$$

or (as $\Delta t \to 0$)

$$\frac{d\varepsilon_i}{dt} = -\frac{1}{1 - \left(1 + e_i(t)\right)W_i(t)} \times$$

$$\times\left(\frac{1}{1 + e_i(t)}\frac{de_i(t)}{dt} + \left(1 + e_i(t)\right)\frac{dW_i(t)}{dt}\right).$$

By inserting the previously derived equality into equation (1), we arrive at

$$\begin{aligned} \frac{dl(t)}{dt} = \int_{l(t)}^{L} \frac{1}{1 - (1 + e(t,\theta,W))W(z,t)} \times \\ \times\left(\frac{1}{1 + e(t,\theta,W)}\frac{de(t,\theta,W)}{dt} + (1 + e(t,\theta,W))\frac{\partial W(z,t)}{\partial t}\right)dz. \end{aligned} \quad (3)$$

To describe the change in the position of the upper movable boundary of the porous medium considering biodegradation processes and the dependencies $e = e(t,\theta,W)$ where Q is the sum of the principal stresses in the porous medium skeleton, we use a kinematic boundary condition. Considering that

$$W(x,t) = \frac{1}{\varrho_{ws}} m_{ws}(x,t) \quad (4)$$

we obtain

$$\frac{dl(t)}{dt} = \int_{l(t)}^{L} \frac{1}{1 - \left(1 + e\left(t,\theta,m_{ws}\right)\right)\frac{m_{ws}(z,t)}{\varrho_{ws}}} \times$$

$$\times\left(\frac{1}{1 + e\left(t,\theta,m_{ws}\right)}\frac{de\left(t,\theta,m_{ws}\right)}{dt} + \frac{\left(1 + e\left(t,\theta,m_{ws}\right)\right)}{\varrho_{ws}}\frac{\partial m_{ws}(z,t)}{\partial t}\right)dz.$$

This condition accounts for non-isothermal conditions, as biodegradation is accompanied by the release of heat, which in turn affects the state of the porous medium. Thus, the kinematic boundary condition allows for the prediction of landfill settlement dynamics, taking into consideration the processes of biodegradation and thermal influence.

III. Modified filtration consolidation equation considering biodegradation

The settlement processes of the surfaces of MSW landfills are determined by two main factors: the consolidation of the porous medium under the influence of its own weight or external loads, and the reduction of the volume of the organic component due to biodegradation. In this case, the consolidation equation for saturated porous media takes the form of [7, equation (12)].

$$\frac{de\left(t,\theta,m_{ws}\right)}{dt}+e\left(t,\theta,m_{ws}\right)\left(\frac{1}{\varrho_p}\frac{d\varrho_p}{dt}-\frac{1}{\varrho_m}\frac{d\varrho_m}{dt}\right)= \\ =(1+e)\frac{\partial}{\partial x}\left(k(e,T)\frac{\partial h}{\partial x}\right),\ x\in\Omega(t)=(l(t);L),\ t>0. \tag{5}$$

Here: ϱ_m – total density of the solid component of the porous medium; ϱ_ϱ – density of the pore fluid; $k(e,T)$- filtration coefficient, which depends on both the temperature T and the void ratio e, which in turn may depend on the mass of organic waste m_{ws} and the sum of principal stresses θ in the porous medium skeleton; h- excess pressure.

The relationship that describes the change in pore pressure in the porous medium requires several assumptions. We assume that changes in stresses in the skeleton of the porous medium and biodegradation processes affect the change in the pore ratio independently. After transitioning to mass for the application of biochemical degradation relationships, the consolidation equation will take the following form:

$$\frac{a\gamma}{1+e}\frac{\partial h}{\partial t}-\beta(1+e)\frac{1}{\varrho_{ws}}\frac{\partial m_{ws}}{\partial t}= \\ =\frac{\partial}{\partial x}\left(k(e,T)\frac{\partial h}{\partial x}\right), \\ x\in\Omega(t)=(l(t);L),\ t>0. \tag{6}$$

IV. Mathematical model of surface settlement of the landfill considering biodegradation

The mathematical model uses a modified filtration consolidation equation supplemented by a kinematic boundary condition for the upper movable boundary of the porous medium.

$$\frac{a\gamma}{1+e}\frac{\partial h}{\partial t}-\beta(1+e)\frac{1}{\varrho_{ws}}\frac{\partial m_{ws}}{\partial t}=\frac{\partial}{\partial x}\left(k(e,T)\frac{\partial h}{\partial x}\right), \\ x\in\Omega(t)=(l(t);L),\ t>0, \tag{7}$$

$$c_T\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)-\varrho c_\varrho u\frac{\partial T}{\partial x}+Q(x,t,T), \\ x\in\ \Omega(t),\ t>0, \tag{8}$$

$$\frac{\partial m_{ws}(x,t)}{\partial t}=-k_{ws}(T)m_{ws}(x,t), x\in\ \bar{\Omega}(t),\ t>0, \tag{9}$$

$$h(x,t)|_{x=l(t)}=\bar{h}_l(t),\ \ t\geq 0, \tag{10}$$

$$T(x,t)|_{x=l(t)}=\bar{T}_l(t),\ t\geq 0, \tag{11}$$

$$u(x,t)|_{x=L}=\left.\left(-k(e,T)\frac{\partial h}{\partial x}\right)\right|_{x=L}=0, t\geq 0, \tag{12}$$

$$\left.\left(-\lambda\frac{\partial T}{\partial x}\right)\right|_{x=L}=0,\ t\geq 0, \tag{13}$$

$$T(x,0)=T_0(x),\ x\in\bar{\Omega}=\left[l_0;L\right], \tag{14}$$

$$m_{ws}(x,t)\Big|_{t=0}=m_0(x),\ x\in\bar{\Omega}(t), \tag{15}$$

$$\frac{dl(t)}{dt}=\int_{l(t)}^{L}\frac{1}{1-(1+e)\frac{m_{ws}}{\varrho_{ws}}}\times \\ \times\left(\frac{a\gamma}{1+e}\frac{\partial h}{\partial t}+(1-\beta)(1+e)\frac{1}{\varrho_{ws}}\frac{\partial m_{ws}}{\partial t}\right)dz; \tag{16}$$

$$l(t)|_{t=0}=l_0<L. \tag{17}$$

Here: h – excess head; k – coefficient of filtration of the porous medium, which depends on temperature T and porosity coefficient, which, in turn, depends on pressures and concentration of biowaste; $m_{ws}(x,t)$ – mass fraction of organic component per unit volume of waste; $n, e=\frac{n}{1-n}$ - porosity and void ratio of the medium; k_{ws} – biodegradation coefficient, dependent on temperature; $m_0(x), h_0(x), \bar{h}_l(t), T_0(x), \bar{T}_l(t)$ - known functions; c_T- volumetric heat capacity of the porous medium; λ - thermal conductivity coefficient of the porous medium; ϱ, c_ϱ – density and specific heat capacity of the pore fluid; $Q(x,t,T)$ – function defining heat generation due to biodegradation; $0<l_0<L$ – specified initial position of the upper movable boundary; a– compression coefficient of the porous medium; γ - specific weight of the pore fluid; u- filtration velocity.

The paper [8] establishes the existence and uniqueness of a classical solution to the boundary value problem for the filtration consolidation equation with a kinematic boundary condition of type (16).

V. Results

In the numerical experiments, we used averaged data, considering the waste storage medium as a two-phase system consisting of pore fluid and the solid phase of the waste itself. It is assumed that the organic waste is uniformly distributed throughout the thickness of the porous medium with an initial total mass of $m_0=300\ \frac{kg}{tone}$.

The numerical experiments showed that the settlement of the landfill surface is significantly affected by temperature. The heat released during biodegradation under non-isothermal conditions leads to greater settlement compared to isothermal conditions. Due to the increase in temperature, after 1000 days, the settlement increased by 7.8%.

TABLE I. SETTLEMENT (IN METERS) IN THE CASE OF AEROBIC BIODEGRADATION PROCESSES

t, days	*Neglecting T*	*Considering T*
50	0.072	0.073
100	0.162	0.164
150	0.252	0.255
200	0.342	0.346
300	0.522	0.532
400	0.702	0.720
500	0.882	0.912
600	1.062	1.107
700	1.242	1.305
800	1.422	1.507
900	1.602	1.712
1000	1.782	1.921

Under high-temperature conditions (up to 83 ℃), biodegradation processes are slowed down due to the reduced biological activity of microorganisms. The results showed that methane production increases in aerobic processes at high temperatures, exceeding the predicted values for isothermal conditions by more than two times.

TABLE II. GAS PRODUCTION DURING AEROBIC BIODEGRADATION

t, days	*neglecting T*	*considering T*
50	39.34	13.82
100	77.83	51.13
150	115.47	89.55
200	152.29	107.43
300	223.53	134.12
400	291.69	156.49
500	356.90	176.59
600	419.29	195.12
700	478.98	212.43
800	536.10	228.73
900	590.74	244.16
1000	643.01	258.84

TABLE III. GAS PRODUCTION DURING ANAEROBIC BIODEGRADATION

t, days	*neglecting T*	*considering T*
50	39.34	7.15
100	77.83	19.68
150	115.47	36.66
200	152.29	58.43
300	223.53	119.92
400	291.69	213.24
500	356.90	339.37
600	419.29	464.84
700	478.98	551.25
800	536.10	617.39
900	590.74	672.89
1000	643.01	721.648

VI. CONCLUSIONS

This work examined the settlement of the surface of MSW landfills, considering the filtration consolidation of porous media. The model allows for the evaluation of the stress-strain state of the landfill without accounting for microbiological activity, while taking into consideration biodegradation and thermal processes.

During the numerical experiments, it was found that accounting for non-isothermal conditions changes the dynamics of surface settlement in the landfill. Thermal processes can lead to both increases and decreases in the predicted settlement values. The results showed that under aerobic conditions, the intensity of biodegradation of organic residues is reduced, and as a result, the settlement of the landfill occurs more slowly than under anaerobic conditions.

The conducted numerical experiments confirm the feasibility of using a mathematical model to predict the settlement of MSW landfills. Considering biodegradation processes allows for the prediction of settlement dynamics and enables the use of calculation data for the design and operation of these facilities.

REFERENCES

[1] Y. Ren, Z. Zhang., M. Huang, "A review on settlement models of municipal solid waste landfills", Waste Management, volume 149, 2022, pp. 79-95. doi: 10.1016/j.wasman.2022.06.019.

[2] M. S. Mousavi, J. Eun, "A Predictive Settlement Modeling Framework Employing Thermal–Hydraulic–Mechanical–Biochemical Processes in Municipal Solid Waste Landfills", International Journal of Geomechanics, volume 23, Issue 6, 04023075, 2023. doi: 10.1061/IJGNAI.GMENG-8115

[3] P. M. Martyniuk, N. V. Ivanchuk, "Effect of the Microorganisms Dynamics on the Base Subsidence of the Solid Household Waste Storage During Consolidation", Journal of Engineering Sciences (Ukraine), 11(1), H21–H28, 2024.

[4] Q. Liu, Y. Deng, T. Wang, "One-dimensional nonlinear consolidation theory for soft ground considering secondary consolidation and the thermal effect", Computers and Geotechnics, volume 104, pp. 22-28, 2018. doi: 10.1016/j.compgeo.2018.08.007.

[5] O.Stepanchenko, L. Shostak, V. Moshynskyi, O.Kozhushko, P. Martyniuk, "Simulating Soil Organic Carbon Turnover with a Layered Model and Improved Moisture and Temperature Impacts", Lecture Notes on Data Engineering and Communications Technologies, 149, pp. 74–91, 2023.

[6] X. Liu, J. Shi., X. Qian, Y. Hu, G. Peng, "One-dimensional model for municipal solid waste (MSW) settlement considering coupled mechanical-hydraulic-gaseous effect and concise calculation", Waste Management, volume 31, Issue 12, 2473-2483, 2011. doi: 10.1016/j.wasman.2011.07.013.

[7] V. A. Herus, P. M. Martyniuk, "Generalization of the soil consolidation equation considering the effects of physicochemical factors", Bulletin of Kharkiv National University named after V. N. Karazin. Series: Mathematical Modeling. Information Technologies. Automated Control Systems, Issue 27, pp. 41–52, 2015.

[8] P. Martyniuk, O. Ostapchuk, T. Tsvietkova, O. Michuta, "Existence and uniqueness of solving boundary problem for quasi-linear parabolic equation with integral condition on free boundary", JP Journal of Heat and Mass Transfer, 15(3), pp. 569–578, 2018.