Application of Quantum Computing in Optimization Problems

https://doi.org/10.31713/MCIT.2024.033

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***Abstract* – The article examines methods for applying quantum computing to solve optimization problems, which are critical in various fields such as logistics, finance, resource management, energy, and others. The potential of quantum algorithms, such as Grover's algorithm and Variational Quantum Algorithms (VQA), is explored for enhancing computational efficiency compared to classical methods. Quantum computing enables significant acceleration of optimal solution search processes and reduces the complexity of problems through parallel computing and quantum superposition.**

***Keywords* – quantum computing, optimization, quantum algorithm, quantum variational algorithm, Grover's search.**

Optimization problems find wide application in various areas of life. In many cases, optimization problems become so complex that classical methods cannot provide a fast or accurate solution. Quantum computing promises to solve this issue by exponentially speeding up certain computational tasks, making it a promising tool for solving optimization problems [1].

Optimization is one of the most common mathematical problems, applied across a wide range of fields: from economics and resource management to route planning in logistics and solving artificial intelligence tasks. Optimization problems involve finding the best possible solution under constrained resources, which requires fast and accurate computation. However, classical optimization methods face with serious limitations as the size of the problem increases, especially in cases where exponential computations are required.

With the development of quantum computing, new methods for solving optimization problems are emerging, potentially providing significant acceleration in the process of finding optimal solutions. [1] Quantum computing is based on principles of quantum mechanics such as superposition, entanglement, and quantum interference, allowing many possible solutions to be processed simultaneously. [2] This enables search and optimization tasks to be solved much faster than with classical computational methods.

Particularly promising are quantum algorithms such as Grover's quantum algorithm, which accelerates the search for optimal solutions, as well as Variational Quantum Algorithms (VQA), which combine classical and quantum computing to improve results in optimization problems. Research and implementation of these quantum algorithms could lead to significant breakthroughs in optimization tasks across various fields where accuracy and speed play a critical role [3].

- The research methodology is based on the application of quantum algorithms for solving optimization problems, which includes both analytical and simulation modeling. The main quantum algorithms used in the study are effective for specific classes of optimization and search problems that exhibit exponential complexity when using classical methods. These algorithms include:
- **Grover's Quantum Algorithm**. This algorithm enables fast searching in unsorted databases or the space of possible solutions. [1] For classical algorithms, searching in an unsorted database requires $O(N)$ operations, whereas Grover's algorithm can accomplish this in $O(\sqrt{N})$ steps. This algorithm serves as the foundation for accelerating problems that involve finding the best solution among many possibilities, such as in combinatorial optimization problems.
- **Variational Quantum Algorithms (VQA)**. One of the most widely used algorithms in this class is the Quantum Approximate Optimization Algorithm (QAOA). This algorithm combines elements of classical computations with quantum computations, providing optimization through a variational principle. [2] As a result, the quantum computing process finds optimal parameters for solving the problem by iteratively improving the quantum state. QAOA is used to solve discrete optimization problems, such as routing, knapsack, and placement problems.
- **Quantum Stochastic Search**. This method is based on quantum fluctuations, allowing for the exploration of a large space of possible solutions for optimization problems in fewer steps compared to classical stochastic methods. This approach is particularly effective for global optimization problems, where it is necessary to find the optimum in a large and complex search space.
- **Hybrid classical-quantum methods.** The research involves the use of both quantum and classical approaches to tackle complex optimization problems. [3] For example, a classical computer system may perform preparatory calculations, while a quantum computer handles the most challenging optimization operations,

such as finding the global minimum of a function. This hybrid model is employed to reduce overall complexity and computation time.

- Simulations of quantum algorithms. To perform modeling and testing of results, specialized software platforms for simulating quantum computations are used, such as IBM Q Experience, Google Cirq, and Microsoft Quantum Development Kit (QDK). [4] These platforms allow for checking and validating the results of quantum algorithms on classical computers, as well as optimizing them before deployment on real quantum systems.

The application of quantum algorithms has demonstrated advantages in solving certain classical optimization problems. For shortest path problems and resource allocation tasks, quantum methods show significant acceleration. [4] Variational quantum algorithms, such as the Quantum Approximate Optimization Algorithm (QAOA), have also shown good results in discrete optimization problems [3].

Quantum computing opens new avenues for solving complex optimization problems that are challenging to address using classical methods. Further research may lead to the implementation of quantum algorithms in practice and significant improvements in results for resource management, logistics, machine learning, and other fields [5].

REFERENCES

[1] Nielsen, M. A., Chuang, I. L. *Quantum Computation and Quantum Information*. Cambridge University Press, 2010.

[2] Farhi, E., Goldstone, J., Gutmann, S., et al. *A Quantum Approximate Optimization Algorithm*, 2014. arXiv:1411.4028 [quant-ph].

[3] Shor, P. W. *Algorithms for quantum computation: discrete logarithms and factoring*. Proceedings 35th Annual Symposium on Foundations of Computer Science, 1994.

[4] Grover, L. K. *A fast quantum mechanical algorithm for database search*. Proceedings of the 28th Annual ACM Symposium on Theory of Computing, 1996.

[5] Montanaro, A. *Quantum algorithms: an overview*. npj Quantum Information, 2, 15023, 2016