Spatial Analogues of Numerical Quasiconformal Mapping Methods for Solving Identification Problems in Anisotropic Media

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Abstract— **The approach to solving the gradient problems of image reconstruction of spatial bodies using applied quasipotential tomographic data that is based on numerical complex analysis methods is extended to cases of anisotropic media. Here the distribution of eigen-directions of the conductivity tensor is considered a priori known. We propose to identify the parameters of the corresponding quasiideal stream by the way of minimizing the functional of the sum of squares of residuals which constructed using differential equations in partial derivatives that relate the quasipotential of velocity and the spatially quasicomplex conjugated stream functions.**

Keywords— applied quasipotential tomography, quasiconformal mappings, identification, numerical methods, anisotropy.

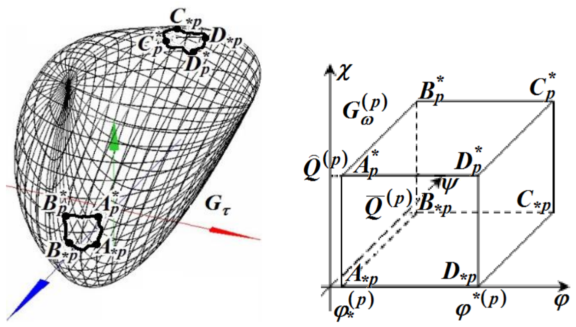
I. Introduction

The scope of electrical tomographic methods for identifying the structure of an anisotropic medium in spatial bodies is becoming more widespread in science and technology. In particular in robotics, geology, medicine, biology, etc. [1]. However, the quality of the corresponding reconstructed images is often less than expected. This is due to the mathematical aspects of restoring the corresponding conductivity coefficient. Particularly widespread is the assumption of the “point-like” surfaces of quasipotential application to the input and output sections [1].

We propose the mathematical formulation of the applied quasipotential tomography problem and the peculiarities of its solving without “point-like” conditions on the sections of the entrance and exit of charges in space, with presence of the anisotropy effect. In this case, the ideas and corresponding algorithms of quasiconformal mapping in anisotropic and three-dimensional cases are described in detail in [2] and [3], respectively.

II. The Applied Quasipotential Tomography Problem

The problem of finding the functions $\varphi^{(p)}=\varphi^{(p)}(\tau)$ (potentials) and $\psi^{(p)}=\psi^{(p)}(\tau)$, $\chi^{(p)}=\chi^{(p)}(\tau)$ (streams) in a single-curved curvilinear body (anisotropic rod, tomographic object) G_τ (fig. 1, a), bounded by a smooth closed surface $\partial G_\tau=\{\tau=(x,y,z):\quad \tau=(\tilde{x}(u,v),\tilde{y}(u,v),\tilde{z}(u,v)),$ $0\le u\le 1,\quad 0\le v\le 1\}$, provided the identification of conductivity tensor $\sigma=\left(\sigma_{\alpha\beta}(\tau)\right)_{\alpha,\beta=1,3}$ is considered.



a) b)

Figure 1. Tomographic object G_τ (a) and corresponding surfaces of complex quasipotential $G_\omega^{(p)}$ (b)

The corresponding statement is as follows [4, 5]:

$$\begin{cases}\sigma(\tau)grad\,\varphi^{(p)}(\tau)=grad\,\psi^{(p)}(\tau)\times grad\,\chi^{(p)}(\tau),\\ grad\,\psi^{(p)}(\tau)\cdot grad\,\chi^{(p)}(\tau)=0,\ \tau\in\tilde{G}_\tau^{(p)};\end{cases}\tag{1}$$

$$\varphi^{(p)}\Big|_{A_{*p}B_{*p}B_p^*A_p^*}=\varphi_*^{(p)},\ \ \varphi^{(p)}\Big|_{C_{*p}D_{*p}D_p^*C_p^*}=\varphi^{*(p)},$$

$$\psi^{(p)}\Big|_{A_{*p}D_{*p}D_p^*A_p^*}=0,\ \ \psi^{(p)}\Big|_{B_{*p}C_{*p}C_p^*B_p^*}=\bar{Q}^{(p)},$$

$$\chi^{(p)}\Big|_{A_{*p}B_{*p}C_{*p}D_{*p}}=0,\ \ \chi^{(p)}\Big|_{A_p^*B_p^*C_p^*D_p^*}=\hat{Q}^{(p)};$$

$$\bar{Q}^{(p)}\hat{Q}^{(p)}=\int\limits_{A_{*p}B_{*p}}\left|\vec{j}^{(p)}\right|ds\int\limits_{A_{*p}A_p^*}\left|\vec{j}^{(p)}\right|ds=Q^{(p)};$$

$$\varphi^{(p)}(M)\Big|_{A_{*p}D_{*p}D_p^*A_p^*}=\underline{\varphi}^{(p)}(M),$$

$$\varphi^{(p)}(M)\Big|_{B_{*p}C_{*p}C_p^*B_p^*}=\bar{\varphi}^{(p)}(M),$$

$$\chi^{(p)}(M)\Big|_{A_{*p}D_{*p}D_p^*A_p^*}=\underline{\underline{\chi}}^{(p)}(M),$$

$$\chi^{(p)}(M)\Big|_{B_{*p}C_{*p}C_p^*B_p^*}=\bar{\bar{\chi}}^{(p)}(M),$$

$$\psi^{(p)}(M)\Big|_{A_{*p}B_{*p}B_p^*A_p^*}=\psi_{**}^{(p)}(M),$$

$$\psi^{(p)}(M)\Big|_{C_{*p}D_{*p}D_p^*C_p^*}=\psi^{**(p)}(M),$$

$$\chi^{(p)}(M)\Big|_{A_{*p}B_{*p}B_p^*A_p^*}=\chi_{**}^{(p)}(M),$$

$$\chi^{(p)}(M)\Big|_{C_{*p}D_{*p}D_p^*C_p^*}=\chi^{**(p)}(M),$$

$$\varphi^{(p)}(M)\Big|_{A_{*p}B_{*p}C_{*p}D_{*p}}=\underset{\smile}{\varphi}^{(p)}(M),$$

$$\varphi^{(p)}(M)\Big|_{A_p^*B_p^*C_p^*D_p^*}=\overset{\frown}{\varphi}^{(p)}(M),$$

$$\psi^{(p)}(M)\Big|_{A_{*p}B_{*p}C_{*p}D_{*p}}=\underset{\smile}{\psi}^{(p)}(M),$$

$$\psi^{(p)}(M)\Big|_{A_p^*B_p^*C_p^*D_p^*}=\overset{\frown}{\psi}^{(p)}(M).$$

Here A_{*p}, B_{*p}, C_{*p}, D_{*p}, A_p^*, B_p^*, C_p^*, $D_p^*\in\partial G_\tau$; $\vec{n}$ is a single vector of the outer normal; M is the point of corresponding section of the surface; $p=\overline{1,\tilde{p}}$; p and $\tilde{p}$ are number and amount of current injections, respectively; functions $\underline{\varphi}^{(p)}(M)$, $\bar{\varphi}^{(p)}(M)$, $\underline{\chi}^{(p)}(M)$, $\bar{\chi}^{(p)}(M)$, $\psi_*^{(p)}(M)$, $\psi^{*(p)}(M)$, $\chi_*^{(p)}(M)$, $\chi^{*(p)}(M)$, $\underset{\smile}{\varphi}^{(p)}(M)$, $\overset{\frown}{\varphi}^{(p)}(M)$, $\underset{\smile}{\psi}^{(p)}(M)$, $\overset{\frown}{\psi}^{(p)}(M)$ and full discharges $Q^{(p)}=\bar{Q}^{(p)}\hat{Q}^{(p)}$ are obtain as results of physical measurements; eigenvalues λ_1, λ_2 and λ_3 of corresponding to (1) matrix we find as follows:

$$\lambda_1=\lambda_1(\tau,...)=\sum_{(0\le\alpha_1+\beta_1+\lambda_1\le s_1)}a_{\alpha_1,\beta_1,\lambda_1}x^{\alpha_1}y^{\beta_1}z^{\lambda_1},$$

$$\lambda_2=\lambda_2(\tau,...)=\sum_{(0\le\alpha_2+\beta_2+\lambda_2\le s_2)}b_{\alpha_2,\beta_2,\lambda_2}x^{\alpha_2}y^{\beta_2}z^{\lambda_2},$$

$$\lambda_3=\lambda_3(\tau,...)=\sum_{(0\le\alpha_3+\beta_3+\lambda_3\le s_3)}c_{\alpha_3,\beta_3,\lambda_3}x^{\alpha_3}y^{\beta_3}z^{\lambda_3},$$

where $a_{\alpha_1,\beta_1,\lambda_1}$, $b_{\alpha_2,\beta_2,\lambda_2}$, $c_{\alpha_3,\beta_3,\lambda_3}$ $(0\le\alpha_1+\beta_1+\lambda_1\le s_1$, $0\le\alpha_2+\beta_2+\lambda_2\le s_2$, $0\le\alpha_3+\beta_3+\lambda_3\le s_3)$ are parameters that are defined during the process of problem solving, and the function of distributions of angles $\theta=\theta(x,y)$ of directions of extremal values of the conductivity coefficient, similar to [4], we considered a priori known.

The iterative algorithm for solving this problem is similar to [4, 5]. One feature is that the minimizing functional is constructed for reasons of providing approximately equal residuals of expressions (1) at each injection.

III. CONCLUSIONS

We propose approach to parameters identification of an anisotropic medium in spatial bodies using applied quasipotential tomographic data that based on the method of identification of the conduction tensor [4] and the algorithm for solving the problem of image reconstruction in curvilinear parallelepiped that bounded by equipotential and stream surfaces. The proposed formulation of the problem allows to take into account the forms of the sections of entrance and exit of the charges; this provides greater mathematical correspondence in comparison to the common methods.

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