# Principle of Least Action in Models and Algorithms of Optimisation States Power System

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*Abstract*— **The possibility and feasibility of applying the principle of least action (PLA) in the optimal control of the states of the electric power system (EPS) is demonstrated. The loss of active power and electricity was taken as the criterion of optimality. The algorithm for determining the optimal state of the EPS is presented. The peculiarity of it is that, in accordance with the PLA, the ideal state of the EPS is first determined, and then parametric constraints are taken into account. The ideal state of the system is characterized by the lowest possible losses, and taking into account active restrictions on the parameters translates the system to the optimal state. The losses increase depending on the heterogeneity of the system in the optimal state of the EPS.**

**Keywords— optimal control, electric power system, principle of least action, minimum of energy losses.**

## I. INTRODUCTION

Defined two problems of optimal control of states of power system. Namely: operative defining of the optimal values of the parameters of system state, providing the minimum criterion of optimality - active power losses; creating of conditions for self-optimization of power system (PS) states by automation. In practice, PS states are constantly perturbed. Consumer loads, power generation, topology, parameters and so on change. Accordingly, the optimal control system should be implemented optimizing the effects that should enter the PS state in the optimality area. It is shown in [1] that this problem can be formulated as a problem of approximation of the current state of the PS to the economic one:

$$F(\mathbf{x},\mathbf{u}) \Rightarrow F(\mathbf{x}_e,\mathbf{u}_e) \qquad (1)$$

provided that

$$\mathbf{x} \in \mathbf{D}_x, \quad \mathbf{u} \in \mathbf{D}_u, \qquad (2)$$

where $\mathbf{x}$, $\mathbf{x}_e$, $\mathbf{u}$, $\mathbf{u}_e$ – parameters of state $\mathbf{x}$ and control parameters $\mathbf{u}$, respectively current and economical states of power system, which are definede value of optimization criterion F параметри режиму $\mathbf{x}$ і параметри керування $\mathbf{u}$; $\mathbf{D}_x$ , $\mathbf{D}_u$ – admissible ranges of x and control parameters $\mathbf{u}$, respectively.

## II. PRINCIPLE OF LEAST ACTION IN MODELS AND ALGORITHMS AS METHOD OF CREATION OF OPTIMIZING INFLUENCE ON POWER SYSTEM STATES

The most cumbersome, given the limitations on the reliability and speed of the solution, are the computable procedures for determining the set of optimal independent PS parameters, especially when it comes to real large-scale power systems.

As shown in [2, 3], PLA determines the optimality of functioning of any system, as well as the development of a system aimed at increasing the level of its ideality. For a system at any moment of its existence, the norm is a qualitative optimum, the depth of which is defined by the level of ideality of the system. This feature of PLA can be used as an idea to create computational process to find the optimal state of PS.

On Fig. 1 are illustrated two approaches to the design of a computational process for determining the optimal system parameters: with using gradient methods and with using PLA.

The essence of gradient methods is that the initial approximation in the domain $x^{(0)}[x_1^{(0)}, x_2^{(0)}]$ of admissible solutions **D** and further, in a particular computational scheme, descends to the optimality region. PLA uses a different algorithm: are defined is ideal PS state of the with lowest possible losses of power in the given conditions, the state of the system $x_e[x_{1e}, x_{2e}]$, which is usually outside the admissible area D; further, the state is introduced into the admissible area from the point $x_e[x_{1e}, x_{2e}]$ to the point $x_o[x_{1o}, x_{2o}]$, in which all restrictions are fulfilled, but the losses are greater. Note that when using gradient methods, the computational process is constructed in the opposite direction - it is using not an ascent but a descent from the level of bigger losses (the point of

initial approach $x^{(0)}[x_1^{(0)}, x_2^{(0)}]$ to the level with smaller losses.

### III. ALGORITHMIZATION OF OPTIMAL CONTROL OF STATES

In Fig. 2 are shown an example of changing the criterion of optimality - power losses in the PS in time and depending on the parameter of the regulating device. In the case of a homogeneous system, the trajectory of the optimality criterion F runs along the bottom of the ravine. This will be always independent on the load, according to the PLA. In other cases, when $X_i/R_i \neq idem$, the trajectory F may pass through the sides of the ravine in any way, depending on the specific operating conditions. However, even in this case, according to the PLA, the power losses will be minimally possible to ensure the technological process. In order to approximate (optimize) the power losses at each point of the trajectory of their change to ideally possible ones, it is necessary to do out optimizing actions in the system constantly during the operation by means of regulation.
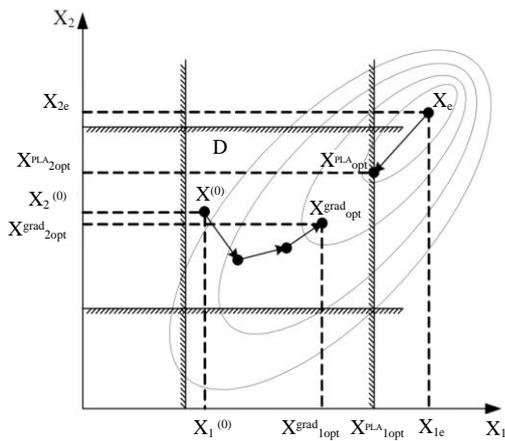


Figure 1.    The power losses levels in system for different values of state parameters
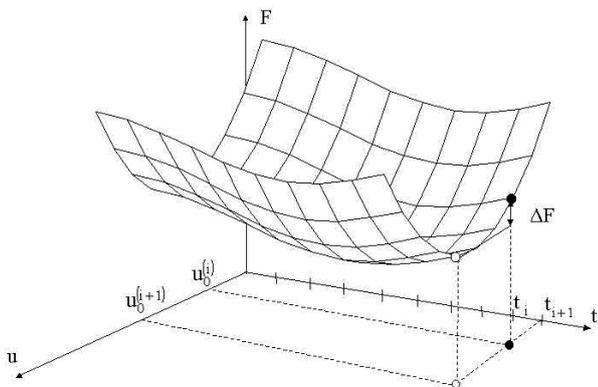


Figure 2.    Changes in the optimality criterion over time and depending on the control parameter

It is possible to compensate for the additional losses in the PS by regulating the voltage at the nodes of the PS and including into the circuits the equalizing electromotive force (EMF). In this formulation of task, the control variables are the EMFs, which must be entered in all closed circuits for optimal current distribution and load of power sources. Equalizing EMFs can be included by changing the transform coefficients of the transformers, which included in the PS circuits.

In practice, the load distribution between active and reactive power sources is performed by algorithms that are based on numerical optimization methods. The disadvantages of these methods with regard to the optimal control of the states of systems, and in particular of PS state, in particular, are discussed in [4, 5]. Therefore, in the framework of tasks (1) - (2), optimization of PS modes is also possible and appropriate to be carried out with the use of PLA.

In Fig. 3 is shown a block diagram of the software complex, which consists of separate blocks: optimization of the PS state by active power; optimization of PS state by reactive power, voltage and transformation coefficients; complex optimization for active and reactive power and voltage.
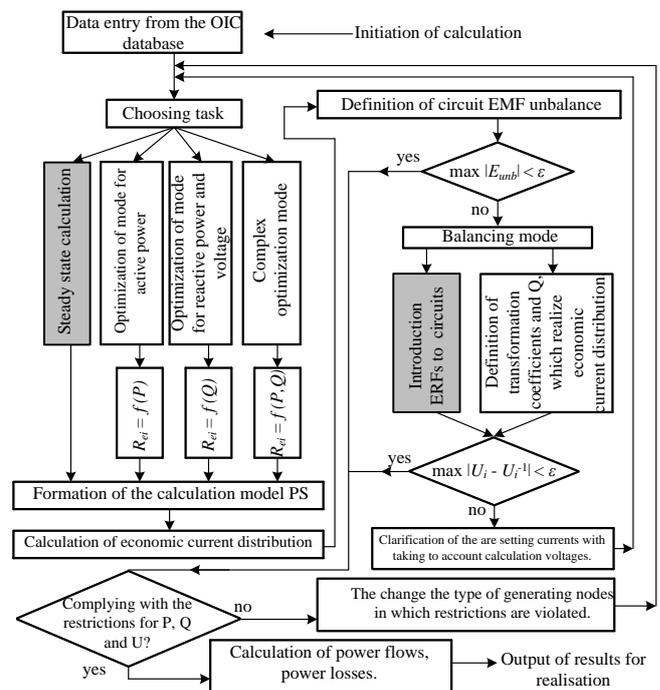


Figure 3.    Changes in the optimality criterion over time and depending on the control parameter

The structural-logical scheme of PC optimization of the PS states looks like this.

1. The problem is selected and the calculation model is formed accordingly. The basic module is the steady-state calculation module, which is carried out according to the following scheme. It is calculated at the given active and reactive power in nodes (except balancing) economic state. Unbalance EMFs are determined and a system of circuit equations is formed, which results in the equalization of EMFs being introduced into the circuits and, after definding the setting currents at the nodes at the calculated voltages, the PS state corresponds to the first and second Kirchhoff laws.

2. If the task is to optimize the state of the PS, then equations are chosen accordingly for the calculation of the economic resistances of $R_{ei}$ sources of active and reactive power [4]:

$$R_{ei} = \frac{B_i(P_i) \cdot U_i^2 \cdot u_i}{P_i^2 \cdot c}, \ R_{ei} = \frac{B_j(P_j) \cdot U_i^2 \cdot Pr_j}{Q_i^2 \cdot c},$$

$$R_{ei} = \frac{B_i(P_i) \cdot U_i^2 \cdot Pr_i}{(P_i^2 + Q_i^2) \cdot c},$$

where $B_i(P_i)$ are consumable characteristics of power sources by optimization of PS of active and reactive powers and complex optimization; $P_i$, $Q_i$ - active and reactive powers of sources; $U_i$ is voltage of nodes; $Pr_i$ - the price of conditional fuel on $i$ a power source; c - the cost of kWh of losses of the electric power.

In these equations, if the power source isnt included in power supply system as subject to managing (it is characteristic of renewable power sources), then instead of consumable characteristics $B_i(P_i)$ take of account the power which is released, and instead of $Pr_i$ these is the holiday cost of kWh of $\beta_i$.

3. The calculation model of the PS is formed and the economic current distribution corresponding to the "ideal" state of the PS is defined. The circuit EMFs of the unbalance are determined, which are converted into transform coefficients of transformers, autotransformers and volt-additional transformers. The introduction of these calculated coefficients of transformation into the PS circuits together with the change in the load of the reactive power sources leads to the balancing of the regime. This optimal state is the closest in the sense of the chosen optimality criterion to the "ideal" state.

4. Due to the fact that the transform and load coefficients of the sources of reactive power (SRP) are discrete quantities, it is possible to break the voltage limits at some nodes. In these cases, the power in the generating units is determined and fixed, leading to constraints. In this case, power losses increase in the PS.

5. The results of the calculation for their realization are displayed. With the "manual" control of the PS state, the results are given as recommendations for operational personnel. If the optimal control state is carried out using the automatic control system (ACS), then the settings of local ACS power sources, transformers and autotransformers.

## IV. Organisation of Computational Process in Personal Computer of Optimisation Power System State

For simplification of logical connections and increase of efficiency of algorithmic realization of the program complex of the analysis and optimization of PS states, the logical scheme of its functioning (fig. 4) is divided into a number of self-sufficient processes: calculation of the normal steady PS state (fig. 4, block A), verification of conditions optimality of the PS mode (Fig. 4, block B), modeling of the ideal state of the PS and correction of independent variables to solve the optimization problem (Fig. 4, block B).
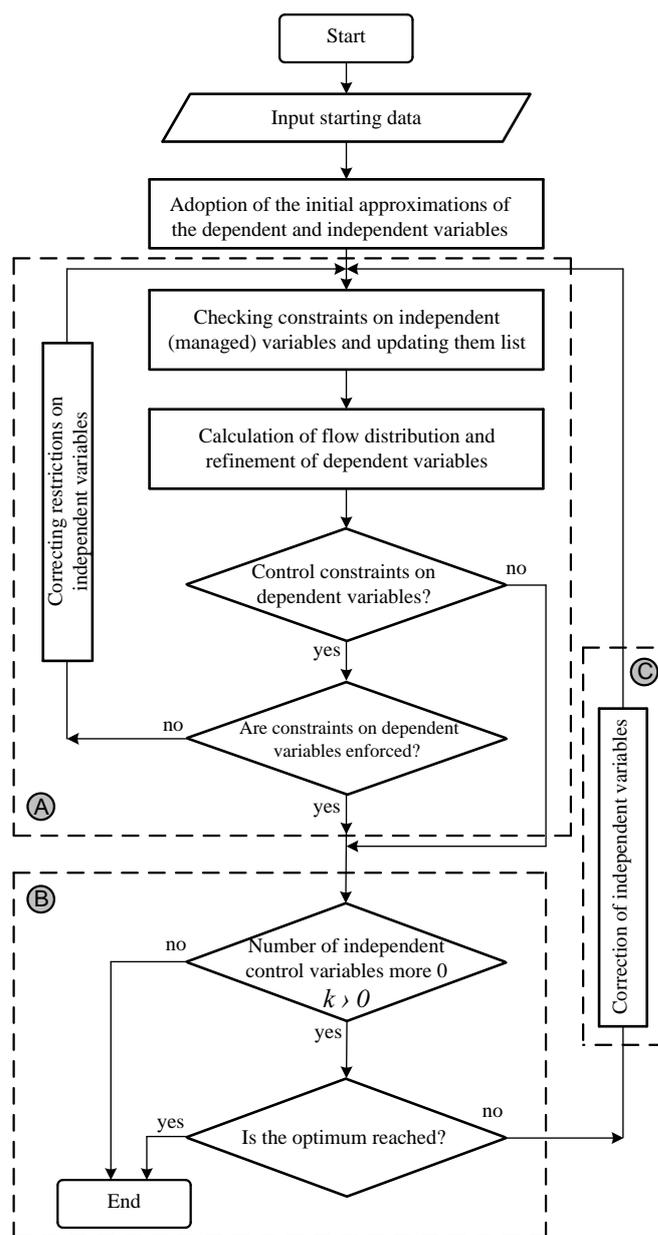


Figure 4. Generalized the block diagram of the algorithm of analysis and optimization of the PS normal state

The current PS state is represented by a set of passive parameters (resistances and conductance's of the PS equivalent schema), as well as dependent and independent variables. Dependent variables include modules $U_i$ and phases $\delta_i$ of voltages in the PS nodes without to the basic and balancing (for the balancing nodes the dependent parameter is $\delta_i$) and the currents loads $I_{ij}$ of the individual elements of the equivalent schema, as well as the derived parameters: power flows $P_{ij}$, $Q_{ij}$, power losses, etc. Independent variables include the voltages of the basic $U_{bo}$, $\delta_{bo}$ and balancing nodes $U_{bi}$, active $P_{G1}$ and reactive $Q_{G1}$ generating powers at the PS nodes of the equivalent schema, longitudinal $k'_{ij}$ and transverse $k''_{ij}$ transformation coefficients of the branches of the equivalent schema (in the case of longitudinal regulating $k''_{ij} = 0$).

Depending on the formulation of the problem of optimization of the PS normal state from the list of independent variables optimized variables are selected, the correction of which allows to enter the normal state in the optimality area by a given criterion.

In the process of solving the problem of optimization of the PS normal state, the constraints on the parameters are controlled in two stages:

- before calculating the normal steady state PS mode (Fig. 4, block A), restrictions on independent (including optimized) variables are controlled, the list of which is defined the optimization problem;

- during the calculation of the normal steady state of the PS (Fig. 4, block A), restrictions on dependent parameters are controlled - voltage modules at independent nodes and modules of currents of branches (active power flows are be controlled for transmission lines, which have voltage 330 kV and more). Constraints are doing by adjusting the constraints (narrowing the ranges of acceptable values) to the corresponding independent optimized variables.

The sequence of simulation of normal and normal-optimal states of the PS is as follows (after initiating the program start manually or automatically by the appropriate means by automatic system of dispatcher control (ASDC)).

Before starting the simulation process, the control of the constraints on independent variables is monitored. In case of violation of constraints, depending on the setting, a corresponding warning-message is formed, or the value of the independent variable is fixed at the limit value, after which it loses the feature optimized. After this, independent variable cannot be considered optimized.

According to the set program parameters in process solving problems of optimization of monitoring dependent parameters it can be carried out with providing, or without ensuring their rated values. In the first case of identification of a deviation of a certain parameter starts iterative process of adjustment of adjacent independent parameters for its introduction to area of permissible values. In other case the corresponding message, and the list of the independent (optimized) variables is formed remains without changes.

The conditions of optimality of the current mode of PS are checked according to the set criterion and the task. The current mode is considered optimal in two cases: if the difference between the optimality criterion on the current and the previous iteration is within a given error (the mode is in the optimality range), or if after checking the constraints on the independent optimized variables they have all been set to the limit values and no optimization signs are found. In the second case, a corresponding message about the insufficiency of the adjustment range for a certain parameter is issued.

## V. CONCLUSIONS

1. Considering the characteristic features of PS as an object of control, as well as the need to implement process control in the context of incomplete and unreliable current information on the state of the object and external influences, it is considered necessary to move to partial decentralization of control using adaptive automatic control systems. At the same time, the use of self-regulating control systems, built on the basis of IPA, allows not to lose the systemic effect of minimizing the total power losses in the electrical networks.

2. The experience that PLA is manifested not only in mechanics, but also in electro mechanics and electrical engineering, indicates the necessity and feasibility of investigating the possibility of using it to construct a method and algorithms for optimal control of states of inhomogeneous electric grids of power systems, when the criterion of optimality is a minimum of power losses. time of its transportation.

Summarizing what has been said about improving the efficiency of optimal control of the normal states of PS and applying for this purpose the principle of least action, it is advisable to explore the possibility of solving the following problems: mathematical modeling of normal states of PS on the basis of PLA for solving the problems of optimal control by the criterion of minimum electricity losses networks during its transportation; development of a method and algorithm for determining the optimal state parameters, the values of which correspond to the minimum power loss in the PS; development of a method and algorithm for determining the parameters of the economic regime of the PS to build a system of optimal control of the PS states.

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