Mathematical Modelling of Mass Transfer in Zones of Complete and Incomplete Saturation under the Action of Systematic Combined Drainage

https://doi.org/10.31713/MCIT.2019.44

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Abstract— **The mathematical model of a processes mass transfer in saturated and unsaturated porous media to the filter-trap in isothermal conditions to the system of vertical drains is presented. The numerical solution of the respective boundary value problem was obtained by the method of finite differences using the numerical method of conformal mappings in an inverse statement.**

Keywords— mathematical model; boundary-value problem; numerical method; numerical conformal mappings; vertical drains; complete and non-complete saturation; salts transfer; filtration; moisture transfer; concentration.

I. Introduction

The flooding of territories takes place as a consequence of the raised level of ground waters. With the aim of fighting flooding are used different hydro ameliorative measures, in particular, the arrangement of drainage systems to discharge excess ground moisture. Drainage systems of a combined type permit to more efficiently influence dangerous hydrodynamic processes and improve resources of soil masses with he objective of their further use. In particular, the use of such soils in agriculture requires the research into the soil water-salt regime. With this aim the mathematical modelling was carried out of the process of salt transfer taking into account the processes of the filtration of salt solutions in the saturated field and moisture transfer in the region of incomplete saturation which take place under the action of combined drainage. The solution of this comprehensive task permitted to study the character of hydrodynamic processes and to forecast them.

II. Problem Statement

The process is considered of mass transfer under the action of systematic combined vertical and horizontal drainage in the field of complete G_1 and incomplete G_2 saturation (fig. 1). There are known heads of fluid H_1 and H_2, concentrations of salts $\tilde{C}_1(x,y,t)$, $\tilde{C}_2(x,y,t)$ in vertical drains, fluid flow H_g in a vertical drain. Distributions are set of salts concentrations in zones of complete and incomplete saturation $\tilde{C}_0^1(x,y)$, $\tilde{C}_0^2(x,y)$ and moisture distribution in the zone of incomplete saturation $\tilde{H}_0(x,y)$.

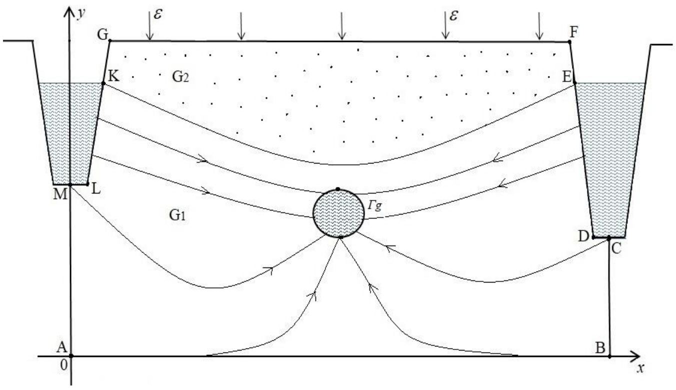

Figure 1. Mass transfer under combined filtration and moisture transfer to systematic combined drainage

It is necessary to calculate distributions of the field of speeds, heads, concentrations of salts in regions of complete and incomplete saturation.

III. Mathematical Model of Processes

The mathematical model of this problem in generally adopted specifications may be described by the following boundary value problem [1, 2, 4-7]:

$$\frac{\partial\left(D_1(c_1)\frac{\partial c_1}{\partial x}\right)}{\partial x}+\frac{\partial\left(D_1(c_1)\frac{\partial c_1}{\partial y}\right)}{\partial y}-\upsilon_x^{/}(c_1)\frac{\partial c_1}{\partial x}-\upsilon_y^{/}(c_1)\frac{\partial c_1}{\partial y}-$$

$$-\gamma_1\left(c_1-C_1^*\right)=\sigma_1\frac{\partial c_1}{\partial t}, \qquad (1)$$

$$\frac{\partial}{\partial x}\left(k_1(c_1,h_1)\frac{\partial h_1}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_1(c_1,h_1)\frac{\partial h_1}{\partial y}\right)-$$

$$-\frac{\partial}{\partial x}\left(\nu(c_1,h_1)\frac{\partial c_1}{\partial x}\right)-\frac{\partial}{\partial y}\left(\nu(c_1,h_1)\frac{\partial c_1}{\partial y}\right)=0, \qquad (2)$$

$$\upsilon_x^{/}\left(c_1\right) = -k(c_1,h_1)\frac{\partial h_1}{\partial x} + \nu(c_1)\frac{\partial c_1}{\partial x},\tag{3}$$

$$\upsilon_y^{/}\left(c_1\right) = -k(c_1,h_1)\frac{\partial h_1}{\partial y} + \nu(c_1)\frac{\partial c_1}{\partial y},$$

$$h_1\big|_{KE} = y\,,\quad \left.\frac{\partial h_1}{\partial n}\right|_{MA\cup BC} = 0\,,$$

$$\left.\frac{\partial h_1}{\partial n}\right|_{AB} = 0\,,\quad h_1\big|_{\Gamma g} = \tilde{H}_g\,,\tag{4}$$

$$h_1\big|_{KLM} = \tilde{H}_1,\; h_1\big|_{DCE} = \tilde{H}_2\,,$$

$$\left.\frac{\partial c_1}{\partial n}\right|_{\Gamma g} = 0\,,\; c_1\big|_{t=0} = \tilde{C}_0^1(x,y)\,,\; \left.\frac{\partial c_1}{\partial n}\right|_{MA\cup AB\cup BC} = 0\,,\tag{5}$$

$$c_1\big|_{KLM} = \tilde{C}_1^1(x,y,t)\,,\; c_1\big|_{EDC} = \tilde{C}_2^1(x,y,t)\,;$$

$$\frac{\partial\left(D_2\left(c_2\right)\frac{\partial c_2}{\partial x}\right)}{\partial x} + \frac{\partial\left(D_2\left(c_2\right)\frac{\partial c_2}{\partial y}\right)}{\partial y} - \upsilon_x\left(c_2\right)\frac{\partial c_2}{\partial x} - \tag{6}$$

$$-\upsilon_y\left(c_2\right)\frac{\partial c_1}{\partial y} - \gamma_2\left(c_2 - C_2^*\right) = \sigma_2\frac{\partial c_2}{\partial t}\,,$$

$$\mu\left(h_2\right)\frac{\partial h_2}{\partial t} = \frac{\partial}{\partial x}\left(k\left(c_2,h_2\right)\frac{\partial h_2}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\left(c_2,h_2\right)\frac{\partial h_2}{\partial y}\right) - \tag{7}$$

$$-\frac{\partial}{\partial x}\left(\nu\left(c_2\right)\frac{\partial c_2}{\partial x}\right) - \frac{\partial}{\partial y}\left(\nu\left(c_2\right)\frac{\partial c_2}{\partial y}\right),$$

$$\upsilon_x\left(c_2\right) = -k(c_2,h_2)\frac{\partial h_2}{\partial x}\,,\; \upsilon_y\left(c_2\right) = -k(c_2,h_2)\frac{\partial h_2}{\partial y}\,,\tag{8}$$

$$h_2\big|_{t=0} = \tilde{H}_0(x,y)\,,\; h_2\big|_{KE} = y\,,\; \left.\frac{\partial h_2}{\partial n}\right|_{EF\cup FG\cup GK} = \varepsilon\,,\tag{9}$$

$$c_2\big|_{t=0} = \tilde{C}_0^2(x,y)\,,\; c_2\big|_{EFGK} = \tilde{C}_1^2\,.\tag{10}$$

Conjugation conditions are set correspondingly for pressure, concentration and salt flows on the saturated and non-saturated domain boundary (depression curves *KE*)

$$h_1\big|_{KE} = h_2\big|_{KE}\,,\; \left.\left(\frac{\partial h_1}{\partial y} - \frac{\partial h_1}{\partial x}\frac{\partial H}{\partial x}\right)\right|_{KE} = 0,\; c_1\big|_{KE} = c_2\big|_{KE}\,,\tag{11}$$

$$\left.\left(D_1(c_1)\frac{\partial c_1}{\partial n} - \nu_1 c_1\right)\right|_{KE} = \left.\left(D_2(c_2)\frac{\partial c_2}{\partial n} - \nu_2 c_2\right)\right|_{KE}.\tag{12}$$

IV. NUMERICAL SOLUTION

The numerical solution of the problem (1)-(12) is found by a method of finite differences using the numerical method of conformic mappings in an inverse statement [3, 4]. Software was created on the basic of developed algorithms and a series of numerical experiments were done.

CONCLUSION

The mathematical modelling was carried out of the process of salt transfer taking into consideration filtrations of salt solutions and moisture transfer under the action of the systematic combined drainage in saturated-non-saturated soil mass.

As a result of the programmic implementation of the developed computation algorithm and the use of numeric conformic mapping a conformic differential network was built and the research was carried out of the process of salts transfer under the combined filtration and moisture transfer in the given field of water saturation. As a result of numeric experiments the distribution is determined of moisture heads, piezometric heads and concentrations of salt solutions taking into account the operation of the combined drainage system which gave the possibility to forecast water-salt regime in these fields.

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