Mathematical Model of Heat-and-Mass Transfer for Steam Humidifier

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Abstract— **The dynamical model of heat-mass exchange for a steam humidifier with lumped parameters, which can be used for synthesis of control systems by inflowing-exhaust ventilation installations, or industrial complexes of artificial microclimate, is considered. A mathematical description that represents the dynamical properties of a steam humidifier concerning the main channels of regulation and perturbation is presented. The resulting dynamical model of a humidification chamber can be the basis for the synthesis of automatic control systems and simulation of transients. A significant ad-vantage of the obtained mathematical model in the state space is the possibility of synthesis and analysis of a multidimensional control system.**

Keywords— dynamical model, state space, steam humidifier

I. INTRODUCTION

Steam humidifying cameras have been widely used in air conditioning. Thanks to technological features, steam humidification is indispensable for creating artificial microclimate: in operating rooms; in the technology of manufacturing drugs, or semi-conductor materials of the electronic industry, and so on. In contrast to the spray-type humidification chambers (the process of adiabatic or polytrophic humidification), in the case of steam humidification (the process of isothermal humidification) there is no need for additional heating of air after moisture [1].

In developing a mathematical model, the task is to determine the limits of its detalization. The dynamical model should be simple for its research and synthesis of the control system, and also take into account the features of heat-mass exchange. Researchers use mathematical models with lumped [2, 3] and distributed [4, 5] parameters to simulate the dynamical processes of heat-mass exchange equipment. Models with lumped parameters are simpler in the calculations and make it possible to obtain an analytical solution. Models with distributed parameters apply for more exact mathematical description. An analytical modeling of equipment with distributed parameters is a rather complicated task, transcendental functions appear in the solution [4]. For such tasks numerical methods of the solution are used in practice.

II. DYNAMICAL MODEL OF STEAM HUMIDIFIER

When developing a mathematical model of a steam humidifier, the model [6] was used as the base model, where the humidification chamber without a steam generator is analyzed. In [6], the differential equation of the humidifier chamber material balance is considered in characteristic of relative humidity, which is linearized. Relative humidity is determined by a significant nonlinear dependence on the temperature of air [1]. As a consequence, the temperature range of the use of the linear model is small. Therefore, the material balance equations of the steam humidification chamber will be considered in characteristic of the air moisture content.

The following simplifications were made during the development of a dynamic model of a steam humidifier: heat exchange with the environment is absent, since the thermal losses of modern heaters do not exceed 5%; the model contains two main dynamic elements with lumped parameters (air space humidifying chamber and steam generator with water); the physical properties of the material flows and the heat transfer surface are brought to the averaged values of the working range.

In the air space of the humidifying chamber with volume $V_A = H \times L \times C$ there is air with temperature $\theta_{A0}(t)$ and moisture content $d_{A0}(t)$, air flow $G_A(t)$. A steam with a mass flow $G_P(t)$ of is fed through the steam pipeline to the humidifying chamber. The steam pipeline is connected to the steam generator. The steam generator contains water, the level of which is automatically maintained by the stabilization system of the level by means of feed water with temperature $\theta_{W0}(t)$ and flow $G_{W0}(t)$. In the steam generator there are electrodes through which the electric current passes by the power $N_E(t)$, at the expense of which the steam is generated. The pair is assimilated in the air, moisturizing it to the output parameters of the air mixture $\theta_A(t)$, $d_A(t)$. Let's consider the heat and material balance for the air mixture of the humidifying chamber and the steam generator.

The heat balance for the air mixture of the humidifying chamber is:

$$G_A\left[c_A(\theta_{A0}-\theta_A)+\frac{r}{1000}(d_{A0}-d_A)\right]+r\,G_P=c_A\,M_A\frac{d\theta_A}{dt}\ ,\ (1)$$

where c_A is heat capacity of the air mixture; r is heat of vaporization; M_A is mass of moist air in volume V_A. Consider the material balance for airspace of the humidification chamber. The moisture content accumulated in the humidifier air space is defined as the difference between the mass input and output pairs

$$\frac{G_A}{1000}(d_{A0}-d_A)+G_P=V_A\frac{d\rho_A}{dt}. \qquad (2)$$

The heat and material balance for the steam generator are represented by the corresponding equations:

$$G_W c_W \theta_{W0}+N_E-r\,G_P=c_W\frac{dM_W\theta_W}{dt}; \qquad (3)$$

$$G_W-G_P=\frac{dM_W}{dt}, \qquad (4)$$

where θ_{W0} is temperature of feed water; θ_W is water temperature in the steam generator (in operating mode $\theta_W=100\ ^oC$); G_P, G_W is mass flows of steam and water; c_W is heat capacity of water; N_E is power of the steam generator; M_W is mass of water in the steam generator. Equations (1) – (4) represent a dynamic model of heat-mass exchange for the steam humidifier of an air conditioner.

The design of the steam generator includes a system for stabilizing the water level. For these reasons $\frac{dM_W}{dt}\approx 0$ and the differential equation (4) becomes algebraic $G_W-G_P\approx 0$. Also, let's take into account the constant temperature of water in the steam generator $\theta_W\approx 100\ ^oC$, from where for equation (3) $\frac{dM_W\theta_W}{dt}\approx 0$, from which it is easy to determine G_P:

$$G_P=\frac{1}{r-c_W\theta_{W0}}N_E. \qquad (5)$$

After grouping similar terms for (1), (2) with regard to (5) and $G_A\approx const$, we obtain a dynamic model of the steam humidifier:

$$\begin{cases} T_A\dfrac{d\theta_A}{dt}+\theta_A=k_0\theta_{A0}+k_1 d_{A0}+k_2 d_A+k_3 N_E; \\ T_d\dfrac{dd_A}{dt}+d_A=k_4 d_{A0}+k_5 N_E; \end{cases} \qquad (6)$$

where $T_A=\frac{M_A}{G_A}$; $T_d=\frac{\omega V_A}{G_A}$; $k_0=1$; $k_1=\frac{r}{1000\,c_A\,G_A}$; $k_2=-k_1$; $k_3=\frac{r}{c_A G_A(r-c_W\theta_{W0})}$; $k_4=1$; $k_5=\frac{1000}{r-c_W\theta_{W0}}$.

The mathematical model (6) is representable in the states space:

$$\mathbf{X}'=\mathbf{AX}+\mathbf{BU}; \qquad (7)$$

where

$$\mathbf{X}=\begin{bmatrix}\theta_A\\ d_A\end{bmatrix};\ \mathbf{A}=\begin{bmatrix}-\dfrac{1}{T_A} & -\dfrac{k_2}{T_A}\\ 0 & -\dfrac{1}{T_d}\end{bmatrix};\ \mathbf{B}=\begin{bmatrix}\dfrac{k_0}{T_A} & \dfrac{k_1}{T_A} & \dfrac{k_3}{T_A}\\ 0 & \dfrac{k_4}{T_d} & \dfrac{k_5}{T_d}\end{bmatrix};\ \mathbf{U}=\begin{bmatrix}\theta_{A0}\\ d_{A0}\\ N_E\end{bmatrix}.$$

III. Conclusions

A dynamic model of heat-mass exchange for a steam humidifier is proposed. In contrast to existing models, mass transfer processes are described by moisture content, which allows us to demarcate the nonlinear effect of temperature on the relative humidity of the air. This approach allows us to rethink the synthesis of the air conditioning control system based on the demarcate between the mutual influence of temperature and humidity.

The model of a steam humidifier is represented by equivalent dependencies: in the form of differential equations (6); in the state space (7). The choice of the mathematical representation is determined by the method of the control system synthesis.

The dynamic model can be used in the design of a digital control system for industrial air conditioners. This will allow the air conditioning control system to be transferred to a qualitatively new level and will ensure the efficient use of energy resources [7].

References

[1] E.M. Belova, “Central Air Conditioning Systems in Buildings,” Euroclimate, Moscow, (2006).

[2] V.V. Vychuzhanin, “Mathematical models of non-stationary modes of air processing in the central ACS,” Bulletin of the Odessa National Maritime University 23, 2007 pp. 172–185.

[3] I.M. Golinko, “Non-stationary Model of Heat-Mass Exchange for a Water Cooler,” In: 12th International Scientific and Practical Conference “Modern problems of scientific support of power engineering”, 2014, pp.137.

[4] I.M. Golinko, N.A. Kubrak, “Modeling and Optimization of Control Systems,” Ruta, Kamyanets-Podilskii, in Ukrainian, 2012.

[5] I.M. Golinko, A.P. Ladanuk, L.D. Koshelieva, “Dynamic Model of the Calorifier Heat Mode,” Information Technology and Computer Engineering 3(16), 2009, pp. 59–63.

[6] V.V. Vychuzhanin, “Scientific and technical bases of operation of ship's central systems of comfortable air conditioning,” dissertation of technical doctor sciences, Odessa, Ukraine, 2009.

[7] I.M. Golinko, “Principles of automatic control systems synthesis for industrial air conditioners,” Automation Technological and Business-Processes. 8, 2016, pp. 33-42.