Mathematical and Computer Simulation of the Interconnected Processes Mass, Heat and Moisture Transfer in Horizontal Soil Media

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Abstract— **The nonlinear mathematical model of a interconnected processes of moisture transfer, mass and heat transfer in horizontal soil layer is presented. The numerical solution of the respective boundary value problem was obtained by the method of finite differences using the monotonic scheme. Software was created on the basic of developed algorithms and a series of numerical experiments were done.**

Keywords—nonlinear mathematical model; boundary-value problem; numerical method; method of finite differences; monotonic scheme; concentration; head; interconnected processes; mass, heat and moisture transfer; aeration zone.

I. Introduction

The important in today's minds there is problem of the security of the population is ecologically clean agricultural products. In this regard, the task of studying the mutual influence of such interconnected processes as moisture transfer, heat and mass transfer in the aeration zone deserves great attention.

Investigations of the moisture transfer process are given partially in [1, 2], and heat and mass transfer - in [1-4]. However, these processes in these works were studied separately, without taking into account mutual influence and interconnection. In this work, the study of the interconnected effect of moisture transfer, mass transfer and heat transfer in the one-dimensional case.

II. Formulation of the Physical Problem

We consider the interconnected processes mass, heat and moisture transfer in horizontal soil layer (Fig. 1). There is one dimensional case. The horizontal soil layer is called of the aeration zone in this case.

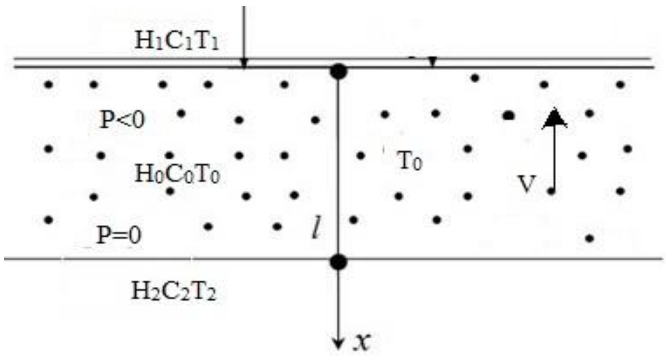

Figure 1. Moisture and heat and mass transfer in non-saturated soil media

At a depth l in the ground is a ground water level. There is a piezometric pressure on the upper and lower surfaces of the unsaturated zone of a soil H_1 and H_2 respectively. The distribution of contaminant concentrations, head and temperature at the initial time $t=0$: $C_0(x)$, $H_0(x)$, $T_0(x)$ are known. The contaminant concentrations $C_1(t)$, $H_1(t)$ and $T_1(t)$ on the upper surface and $C_2(t)$, $H_2(t)$, $T_2(t)$ on the level of subsoil water are also known.

It is necessary to build the adequate mathematical model, find numerical solution and develop software algorithm for further investigation of the distribution moisture, head, concentrations and heat distribution $\Theta(x,t)$, $h(x,t)$, $c(x,t)$, $T(x,t)$ on the moisture transfer area at a given time steps.

III. Mathematical Model

Transfer of salts dissolved in water and heat by moisture transfer occurs under the influence of the pressure gradients of moisture, the concentration of salts and the temperature. The interconnected processes moisture transfer, mass and the heat transfer proceed in accordance with the generalized Darcy's-Kluta's and Fick's and Four's laws.

The nonlinear mathematical model of this problem in generally adopted specifications may be described by the following boundary value problem [1-7]:

$$\frac{\partial}{\partial x}\left(D\left(c,\Theta,T\right)\frac{\partial c}{\partial x}\right)-v\ (c,\Theta,T)\frac{\partial c}{\partial x}-\gamma\left(c-C^{*}\right)+\frac{\partial}{\partial x}(D_{T}\frac{\partial T}{\partial x})=\frac{\partial(\Theta c)}{\partial t},$$

$$\frac{\partial}{\partial x}(K(c,h,T)\frac{\partial h}{\partial x})-\frac{\partial}{\partial x}\left(\upsilon\frac{\partial c}{\partial x}\right)-\frac{\partial}{\partial x}\left(\upsilon^{T}\frac{\partial T}{\partial x}\right)+f=\mu(h)\frac{\partial h}{\partial t},$$

$$\frac{\partial}{\partial x}(\lambda_{T}\frac{\partial T}{\partial x})-\rho C_{\rho}v\frac{\partial T}{\partial x}=C_{T}\frac{\partial T}{\partial t},\quad x\in(0;l),\ \ t>0,$$

$$v=-k(c,h,T)\frac{\partial h}{\partial x}+\upsilon\frac{\partial c}{\partial x}+\upsilon_{T}\frac{\partial T}{\partial x},\ \ x\in(0;l),t>0,$$

$$c(x,0)=C_{0}(x),\quad l_{1}c(0,t)=C_{1}(t),$$

$$l_{2}c\left(l,t\right)=C_{2}(t),\ x\in(0;l),\ t>0,$$

$$h(x,0)=H_{0}(x),\quad h(0,t)=H_{1}(t),\quad h(l,t)=H_{2}(t),\ \ x\in(0;l),$$

$$t>0,$$

$$T(x,0)=T_{0}(x)$$

$$T(0,t)=T_{1}(t),\quad T(0,t)=T_{2}(t),$$

IV. Numerical Solution

The numerical solution of this problem is found by a method of finite differences using the monotonic scheme [8]. Software was created on the basic of developed algorithms and a series of numerical experiments were done. As a result of the programming implementation on C++ of the problem the distribution was found of the fields of the concentration of salt and heat and moisture.

Conclusion

The problem of non-isothermal transfer of moisture and salts in natural disperse media (aeration zone) is formulated. The nonlinear mathematical model of a interconnected processes of moisture transfer, mass and heat transfer in horizontal soil layer is presented. The numerical solutions of the complicated nonlinear boundary value problem have been proposed. In a series of numerical experiments, the influence of mass and heat transfer processes on moisture transfer and vice versa is analyzed.

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