

Mathematical Modeling of Pollution Transfer in Open Channel

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Abstract— The problem of pollution transfer by water flow in open channel was considered. The mathematical model of the problem was constructed. The numerical solution of the one-dimensional boundary problem was obtained. The computational algorithm for solving the problem was programmed to implement. A series of numerical experiments with their further analysis was conducted.

Keywords— mathematical modeling, boundary problem, open channel, mass transfer, monotone difference scheme, velocity of fluid flow.

I. INTRODUCTION

For the prediction of the state and quantitative assessment of hydro-ecological systems, their reactions to anthropogenic impact in large areas, the method of mathematical modeling is extremely effective. The method of mathematical modeling is important especially when the result is not only a description and assessment of the state of the aquatic environment, but also its regulation taking into account the hydro reserve, which should ultimately serve for optimizing the relationship of human society with the aquatic environment [1, 2].

Modeling of processes of water pollution can be realized with the help of various mathematical models of pollutants distribution, differences between which are caused by features of water objects, different physical and chemical properties of pollutants.

Various models are used to describe the spreading of pollutants in the aquatic environment: chamber, imitation and optimization ones. The models are based on partial differential equations. They can simulate conditions of forming quality and quantity of water (or separate processes that determine the quality of water). The class of physical processes studied by numerical modeling of finite difference methods covers almost all processes described by partial differential equations. The process of pollution transfer in reservoir is no exception.

We have considered the problem of pollution spreading in one particular case: in open channel. And we propose to solve this problem with help monotone difference scheme, that can be easier than existing ways for solving this problem.

II. THE PROBLEM OF SPREADING POLLUTION IN OPEN CHANNEL

The problem of pollution transfer in open channel for viscous incompressible fluid was considered (Fig. 1).

The length of the channel is l . Since the length of the channel is much larger than its height, the task is considered one-dimensional. In the area of water movement throughout the channel there is some change in the elevation above the sea level and this change occurs gradually in some one direction.

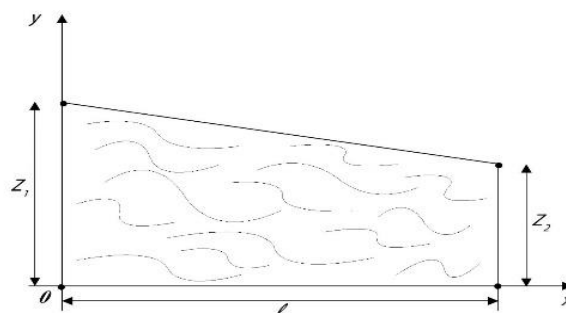


Figure 1. Pollution transfer in open channel

It is known the distribution of the concentration $\tilde{C}_0(x)$ of pollutants in the channel at the initial time $t = 0$ as well as the concentration of pollutants $\tilde{C}_1(t)$, $\tilde{C}_2(t)$ respectively, on the left and right boundaries of the channel.

The process of solute transfer in the open channel is in accordance with the Navier-Stokes equation.

The following physical processes are present in this problem: the process of fluid motion and the process of convective-diffusion pollution transfer.

According to the equations describing these processes [3-5] the mathematical model will look like this:

$$D \frac{\partial^2 c}{\partial x^2} - V \frac{\partial c}{\partial x} - \gamma(c - C_*) = \frac{\partial c}{\partial t}, \quad 0 \leq x \leq l, \quad 0 < t \leq t_1, \quad (1)$$

$$\rho \frac{\partial V}{\partial t} + \rho \frac{\partial V}{\partial x} V + \frac{\partial P}{\partial x} = \mu \frac{\partial^2 V}{\partial x^2}, \quad 0 \leq x \leq l, \quad 0 < t \leq t_1, \quad (2)$$

$$\rho \frac{\partial V}{\partial x} = 0, \quad 0 \leq x \leq l, \quad (3)$$

$$c(x, 0) = \tilde{C}_0(x), \quad 0 \leq x \leq l, \quad (4)$$

$$l_1 c(0, t) = \tilde{C}_1(t), \quad 0 < t \leq t_1, \quad (5)$$

$$l_2 c(l, t) = \begin{cases} c(l, t) = \tilde{C}_2(t), \\ \frac{\partial c(l, t)}{\partial x} = 0, \end{cases} \quad 0 < t \leq t_1, \quad (6)$$

$$V(x, 0) = \tilde{V}_0(x), \quad 0 \leq x \leq l, \quad (7)$$

$$V(0, t) = \tilde{V}_1(t), \quad 0 < t \leq t_1, \quad (8)$$

$$V(l, t) = \tilde{V}_2(t), \quad 0 < t \leq t_1. \quad (9)$$

Here D – diffusion convection coefficient; $c(x, t)$ – concentration of solutes at a point x at time moment t ; V – velocity of fluid motion; γ – mass transfer rate factor; C_* – concentration of boundary saturation; ρ – density of fluid; P – pressure of fluid; μ – fluid viscosity; t_1 – the length of time the process is studied; $l_i, i = \overline{1, 2}$ – operators that set boundary conditions at the channel boundaries.

Equation (3) we can write like that:

$$\frac{\partial V}{\partial t} = \nu \frac{\partial^2 V}{\partial x^2} - \frac{\partial V}{\partial x} V - \frac{1}{\rho} \frac{\partial P}{\partial x}, \quad (10)$$

Here $\nu = \frac{\mu}{\rho}$ – kinematic viscosity of water.

Since the piezometric pressure in the open channel is zero, the pressure will be determined as follows:

$$p = \frac{Z_1 \tilde{\gamma} - Z_2 \tilde{\gamma}}{l} x - Z_1 \tilde{\gamma}, \quad 0 \leq x \leq l. \quad (11)$$

So

$$\frac{\partial P}{\partial x} = \tilde{\gamma} (Z_1 - Z_2). \quad (12)$$

Then given (12), equation (10) takes the form:

$$\frac{\partial V}{\partial t} = \nu \frac{\partial^2 V}{\partial x^2} - \frac{\partial V}{\partial x} V - \frac{1}{\rho} \tilde{\gamma} (Z_1 - Z_2). \quad (13)$$

A numerical solution is obtained for a one-dimensional boundary value problem (1)-(9) using the finite difference method. For boundary problems (1), (4)-(6) and (13), (3), (7)-(9) corresponding monotone schemes were constructed [6, 7]. The solution to each of these problems was found by the sweep method.

A computational algorithm was also programmatically implemented. A significant number of numerical experiments were carried out on the basis of a software implementation of a computational algorithm for solving the problem of pollution transfer in open channel when setting boundary conditions of the first and second kinds for various input data that reflect the physicochemical properties of the water flow and soil.

III. CONCLUSIONS

Based on the results of numerical experiments, graphical representations of fluid velocity distributions and pollutant concentrations in the channel were obtained, as well as their numerical values in the form of tables.

Analyzing the results, we can draw the following conclusions: the level of pollutants concentration is influenced by certain physicochemical properties of water flow of different boundary conditions, as well as the distribution of the velocity of fluid motion.

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