Mathematical Modeling of the Process of Micro Irrigation in the Soil Layer under Conditions of Heat and Mass Transfer

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Anatolii Vlasyuk
Department of Economics, Mathematical Modeling and Information Technologies
The National University of Ostroh Academy
Ostroh, Ukraine
Anatoliy.Vlasyuk@oa.edu.ua

Viktor Ogiychuk
Department of Economics, Mathematical Modeling and Information Technologies
The National University of Ostroh Academy
Ostroh, Ukraine
Viktor22101@gmail.com

Abstract— **The nonlinear mathematical model of a process micro irrigation in non-saturated of soil layer under of heat and mass transfer has presented. The numerical solution of the respective boundary value problem has obtained by the method of finite differences using the monotonic scheme. Software had created on the basic of developed algorithms and a series of numerical experiments were done.**

Keywords—nonlinear mathematical model; boundary-value problem; numerical method of finite differences; monotonic scheme; moisture; heat and mass transfer; micro irrigation, ground water level; wetting front.

I. Introduction

Currently, irrigation problems, in particular micro irrigation, deserve more and more attention in water management practice. In this regard, the task of studying the mutual influence of such interconnected processes as moisture transfer, heat and mass transfer in the aeration zone deserves great attention.

Partially, the process of micro irrigation in soils had studied in [1]. Investigations of the moisture transfer process are given partially in [2, 3], and heat and mass transfer - in [4, 5]. However, these processes in these works were studied separately, without taking into account mutual influence and interconnection. In this work, the study of the interconnected effect of moisture transfer, mass transfer and heat transfer in the one-dimensional case.

II. Formulation of the Physical Problem

Let micro irrigation had performed on the surface of the soil. We consider the process of micro irrigation in non-saturated of soil layer under condition of heat and mass transfer in horizontal soil layer (Fig. 1) (one-dimensional case). In this case, the soil layer is called the aeration zone.

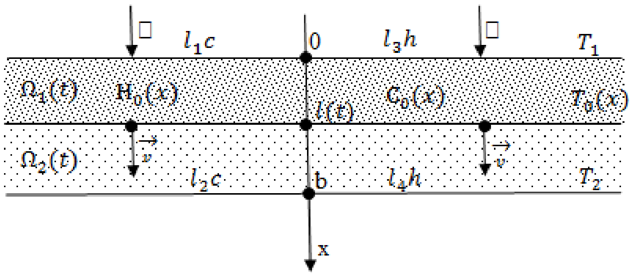

Figure 1. Process of micro irrigation in non-saturated of soil layer under condition of heat and mass transfer

At a depth b in the ground is a wetting front. At a depth l in the ground is a ground water level. There is a moisture pressure on the upper and lower surfaces of the unsaturated zone of a soil H_1 and H_2 respectively. The distribution of contaminant concentrations, head and temperature at the initial time $t = 0$: $C_0(x)$, $H_0(x)$, $T_0(x)$ are known. The contaminant concentrations $C_1(t)$, $H_1(t)$ and $T_1(t)$ on the upper surface and $C_2(t)$, $H_2(t)$, $T_2(t)$ on the level of subsoil water are also known.

It is necessary to build the adequate mathematical model and finite difference scheme for the implementation of the corresponding boundary value problem, find numerical solution and develop software algorithm for further investigation of the distribution moisture, head, concentrations and heat distribution $\Theta(x,t)$, $h(x,t)$, $c(x,t)$, $T(x,t)$ on the moisture transfer area at a given time steps. It is also necessary to find the position of the wetting front at any time.

III. Mathematical Model

Transfer of salts dissolved in water and heat by moisture transfer occurs under the influence of the pressure gradients of moisture, the concentration of salts and the temperature. The interconnected processes moisture transfer, mass and the heat transfer proceed in accordance with the generalized Darcy's-Klyuta's and Fick's and Four's laws.

The mathematical model of this problem in generally adopted specifications in domains may be described by the following boundary value problem [1-9]:

$$\frac{\partial\left(D_T(c_i)\frac{\partial c_i}{\partial x}\right)}{\partial x} - v_i\frac{\partial c_i}{\partial x} - \gamma_i(c_i - C_i^*) + D_{T_i}\frac{\partial^2 T_i}{\partial x^2} = \frac{\partial(\Theta_i c_i)}{\partial t},$$

$$\mu_i(h)\frac{\partial h_i}{\partial t} = \frac{\partial}{\partial x}\left(k_i(c_i,h_i,T_i)\frac{\partial h_i}{\partial x}\right) - \upsilon_i\frac{\partial^2 c_i}{\partial x^2} - \upsilon_i^T\frac{\partial^2 T_i}{\partial x^2},$$

$$\lambda_i\frac{\partial^2 T_i}{\partial x^2} - \rho_i v_i c_p^i\frac{\partial T_i}{\partial x} = c_T^i\frac{\partial T_i}{\partial t},$$

$$v_i = -k_i(c_i,h_i,T_i)\frac{\partial h_i}{\partial x} + \upsilon_i(c_i)\frac{\partial c_i}{\partial x} + \upsilon_i^T\frac{\partial T_i}{\partial x},$$

$$c_1(0,t) = \widetilde{C_1}(t), c_2(l,t) = C_2(t), c_i(x,0) = C_0^i(x),$$

$$h_i(x,0) = H_0^i(x),\ h_1(0,t) = H_1(t), \qquad h_2(l,t) = H_2(t),$$

$$T_i(x,0) = T_0^i(x),\ T_1(0,t) = T_1(t),\ T_2(l,t) = T_2(t),$$

$$[c] = [T] = \left[vc - D\frac{\partial c}{\partial x}\right] = [h] = 0, x = l(t).$$

IV. Numerical Solution

The numerical solution of this problem is found by a method of finite differences using the monotonic scheme [10]. Software was created on the basic of developed algorithms and a series of numerical experiments were done. As a result of the programming implementation on C++ of the problem the distribution was found of the fields of the concentration of salt and heat and moisture.

Conclusion

The problem of micro irrigation processes of in non-saturated of soil layer under condition of heat and mass transfer in horizontal soil layer is formulated. The nonlinear mathematical model of a interconnected processes of moisture transfer, mass and heat transfer in horizontal soil layer is presented. The numerical solutions of the complicated nonlinear boundary value problem have been proposed. In a series of numerical experiments, the influence of mass and heat transfer processes on moisture transfer and vice versa is analyzed. As a result of the numerical solution tasks found the position of the front wetting at any given time.

Based on the conducted numerical experiments, a prediction is made about soil cleansing over time.

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